# DYNAMIC PROPERTIES INVESTIGATION ABOUT CONICAL AMB

Yang Jing Xi'an Jiaotong University, Shanxi xi'an, Post Code 710048, China Yjzhd @163.com Gen Hai Peng, Yu Lie Xi'an Jiaotong University, Shanxi xi'an, Post Code 710048, China

#### ABSTRACT:

Bearing dynamic properties can influence the properties of bearing-rotor system directly. In this paper thirty dynamic property coefficients are defined and deduced to describe the dynamic properties of conical active magnetic bearing. When the system is in balanced position, because of some errors in manufacturing or fixing, some deviation movements  $(x, y, z, \varphi, \psi)$  occur between the rotating center relative to the bearing center, so the bearing radial clearances are in uneven distribution. The variations of the dynamic properties caused by the deviation  $(x, y, z, \varphi, \psi)$  are analysised.

### 1. INTRODUCTION

Because of no contact between the rotor and the housing, magnetic bearings have no friction and low losses, and the dynamic properties of the bearing can be controlled and adjusted. Active magnetic bearing (AMB) is widely used in high speed rotating machinery [1]. This paper studied conical AMB, bearing iron core has a angle  $\beta$ , so bearing controlling forces are disassembled into radial force and axial force. For five freedom degrees conical AMB system, task of supporting and controlling rotor can be carried out by a pair of conical bearing Without axial bearings, heat produced by eddy currents is reduced. In the mean time, the span of the rotor can be reduced too, this structure has advantages in improving the stiffness of bearings, so as to high speed rotating machinery. In particular, conical AMB is very useful in precision positioning devices. In 1989, Inoue and Shimomura used conical bearing in controlling poses of out space flywheel. Jeong, Kim

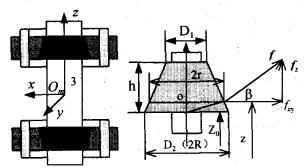


FIGURE 1: Structure of conical AMB

used conical bearing in designing joints of robot. C, W. Lee, and H. S. Jeong (1996) studied the dynamic modleing and optimal control of cone-shaped active magnetic bearing systems. Mohamed and Emad (1992) suggested a nonlinear model for the conical magnetic bearing in the state variable form, and simulated the controlled system with a controller designed using Q-parameterization theory. However, all these research were about system model and controlling [2,3,4,5], there has been littel investigation on the dynamic properties. But it is necessary to investigate the dynamic properties of conical AMB.

In this paper, a total of 30 rotor dynamic coefficients are defined to describe the dynamic properties of conical AMB systematically. These coefficients can be caculated by a general method presented in this paper and can be used for the coupled dynamic analysis.

#### 2. BEARING CLEARANCE VARIATIONS

A conical AMB system is be shown in Figure 1. When any deviation  $(x, y, \varphi, \psi)$  occurred from rotor

center to bearing geometry center, The bearing clearance could be in uneven distribution [6].

As shown in figure 1, when a deviation z is taken in Z direction, the variety of clearances in bearing's normal direction at any positions are:

$$\Delta \delta_1 = \pm (-X\psi + Y\varphi + z)\sin \beta$$

Where:  $X = r \cos \phi$ ,  $Y = r \sin \phi$ ,

$$r = R \mp (z_b - z_0) t g \beta; \quad 0 < \phi \le 2\pi;$$

for the upper bearings:  $z_0 < z_b < z_0 + h$ ,;

for the down bearings:  $-(z_0 + h) < z < -z_0$ ;

When a deviation  $(x, y, \varphi, \psi)$  is taken place in XY plane (as shown in figure 1), the variety of clearances in bearing's normal direction is:

$$\Delta \delta_2 = (x \cos \phi + y \sin \phi) \cos \beta$$

among the Equ: 
$$x^* = x + z\psi$$
;  $y^* = y - z\varphi$ ;

thereout, the clearances in bearing's normal direction at any positions are:

$$\delta = \delta_0 + \Delta \delta_1 - \Delta \delta_2$$

### 3.DYNAMIC COEFFICIENTS OF CONICAL AMB

At first, some hypotheses are given as followed:

- (1) Bearing forces are perpendicular to the bearing faces.
- (2) All of loads are beard on whole width of bearing.
- (3) Take no account of flux leaking or fringing phenomenon.

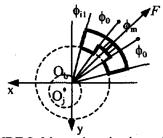


FIGURE 2: Magnetic pole nip angle AMB

A pair of magnetic pole in conical AMB is been shown in Fig.2. Applying Ampere's law to the magnetic path, electromagnetism forces in per unit area of magnetic pole are:

$$f_1 = f_2 = \frac{B^2}{2\mu_0} = \frac{\mu_0(Ni)^2}{2(\delta_1 + \delta_2)^2} \tag{1}$$

(N is number of turns per magnetic pole; i is coil current;  $\mu_0$  is air magnetic conductance;  $\delta_1, \delta_2$  are clearances of magnetic pole),

The total force produced by a pair of magnetic pole can be obtained by intergrating:

$$F_{k} = \int_{t_{0}}^{t_{0}+h} \int_{k_{1}}^{\phi_{k_{1}}+\phi_{0}} f_{1} dz d\phi + \int_{t_{0}}^{t_{0}+h} \int_{k_{1}+2\phi_{0}+\phi_{m}}^{\phi_{k_{1}}+\phi_{0}+\phi_{m}} f_{2} dz d\phi ;$$

It can be written in x, y, z directions

$$F_{k} = iF_{k}\cos\phi + jF_{k}\sin\phi + kF_{k}tg\beta;$$

Whereby, forces and moments produced by a pair of magnetic poles in conical AMB can be expressed as:

$$F_{xk} = \int_0^{\epsilon_0 + h} \int_{\lambda_1}^{\phi_{k_1} + \phi_0} f_1 r_b \cos \phi ddz d\phi$$

$$+ \int_0^{\epsilon_0 + h} \int_{\lambda_1 + 2\phi_0 + \phi_m}^{\phi_{k_1} + \phi_0 + \phi_m} f_2 r_b \cos \phi dz d\phi$$
(2)

$$F_{yk} = \int_{t_0}^{t_0+h} \int_{t_{k+1}}^{t_{k+1}+\phi_0} f_1 r_b \sin \phi ddz d\phi + \int_{t_0}^{t_0+h} \int_{t_{k+1}+\phi_0+\phi_m}^{t_{k+1}+\phi_0+\phi_m} f_2 r_b \sin \phi dz d\phi$$
(3)

$$F_{zk} = \int_{0}^{e_0 + h} \int_{\phi_{k1}}^{\phi_{k1} + \phi_0} f_1 r_h tg \beta dz d\phi$$

$$+ \int_{0}^{e_0 + h} \int_{\phi_{k1} + 2\phi_0 + \phi_m}^{\phi_{k1} + \phi_0 + \phi_m} f_2 r_h tg \beta dz d\phi$$
(4)

$$M_{xk} \approx \int_{t_0}^{t_0+h} \int_{k_1}^{\phi_{k_1}+\phi_0} f_1 r_b Y_a tg \beta dz d\phi$$

$$+ \int_{t_0}^{t_0+h} \int_{k_1+2\phi_0+\phi_m}^{\phi_{k_1}+\phi_0+\phi_m} f_2 r_b Y_a tg \beta dz d\phi$$
(5)

$$M_{yk} \approx -\int_{z_0}^{z_0+h} \int_{y_{k1}}^{\phi_{k1}+\phi_0} f_1 r_b X_a tg \beta dz d\phi$$

$$-\int_{z_0}^{z_0+h} \int_{y_{k1}+\phi_0+\phi_m}^{\phi_{k1}+\phi_0+\phi_m} f_2 r_b X_a tg \beta dz d\phi$$
(6)

For a magnetic bearing with N pairs of magnetic the load capacity force  $F_s$  and moment  $M_s$  can be

obtained: 
$$F_{s} = \sum_{k=1}^{n} F_{sk}$$
,  $(s = x, y, z)$ ;  
 $M_{s} = \sum_{k=1}^{n} M_{sk}$ ,  $(s = x, y)$ ;

According to equations (2~6), the load capacity of conical AMB are not only have relations to bearing structure, bearing winding current, but also have relations to relatively position between rotor and bearing, whereby position stiffness and current stiffness coefficients are introduced to describe the dynamic properties of conical AMB.

The position stiffness matrix can be defined as:

$$k_{s} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} & k_{x\varphi} & k_{x\psi} \\ k_{yx} & k_{yy} & k_{yz} & k_{y\varphi} & k_{y\psi} \\ k_{zx} & k_{zy} & k_{zz} & k_{z\varphi} & k_{z\psi} \\ k_{xx}^{m} & k_{xy}^{m} & k_{xz}^{m} & k_{x\varphi}^{m} & k_{x\psi}^{m} \\ k_{yx}^{m} & k_{yx}^{m} & k_{yz}^{m} & k_{y\varphi}^{m} & k_{y\psi}^{m} \end{bmatrix};$$
 (7)

Current stiffness matrix can be defined as:

$$k_{i} = [k_{yi}, k_{yi}, k_{zi}, k_{xi}^{m}, k_{yi}^{m}]^{T};$$
(8)

thereinto 
$$k_{ps} = \sum_{k=1}^{n} \frac{\partial F_{p}}{\partial s}$$
;  $k_{qs}^{m} = \sum_{k=1}^{n} \frac{\partial M_{q}}{\partial s}$ ;

$$k_{pi} = \sum_{k=1}^{n} \frac{\partial F_{p}}{\partial i}; \quad k_{qi}^{m} = \sum_{k=1}^{n} \frac{\partial M_{q}}{\partial i};$$

$$(s=x,y,z,\varphi,\psi;p=x,y,z,q=x,y);$$

### **4.NUMERICAL RESULTS**

### 4.1 Influence of Deviation On The Static properties

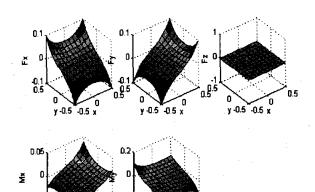


FIGURE 3: Variation in load capacity  $F_x$ ,  $F_z$  and  $M_x$  for deviation x, y

TABLE 1: Parameters of conical AMB

| Name                                | Dimension value       |
|-------------------------------------|-----------------------|
| Rotor diameter                      | D0=36mm               |
| Diameter of small end               | D1=58.4mm             |
| Diameter of big end                 | D2=67.6mm             |
| Distances between two big end plane | 2z <sub>0</sub> =75mm |
| Width of bearing                    | h <sub>0</sub> =35mm  |
| Working clearances                  | $\delta_0 = 0.2$ mm   |
| Number of magnetic pair poles       | n=4                   |
| Winding circles                     | N= 200                |
| Coils current                       | I=4                   |
|                                     |                       |

Table 1 gives the parameters a conical AMB. The static load capacity and the dynamic stiffness coefficients can be calculated with non-dimension.

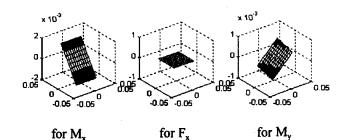


FIGURE 4: Variation in load  $F_{x}$ ,  $M_z$  and  $M_y$  for various tilting angles

The static properties of AMB discussed in this paper are the maximum load capacity. Influence of deviation on the system static capacity can be calculated using equ  $(2\sim7)$ . Fig. 3 and Fig. 4 show the influence of the parallel deviation (x, y) and the tilting angle deviation  $(\varphi, \psi)$  on the system static properties respectively.

- (1) Influence of the deviation x on  $F_x$  (the carrying capacity in X direction) are very large, when the direction of the rotor deviation is in accordance with the load, the load capacity of the bearing decreases; By contraries, the load capacity of the bearing increases.
- (2) Influence of the deviation y on  $F_x$  are very small, which indicate the coupling of the bearing between X and Y are very small;
- (3) Influence of the deviation z on the  $F_z$  (load

capacity in Z direction) are very small;

- (4) Influence of the tilting angle deviation  $(\varphi, \psi)$  on the load moment  $M_s$  is very large. When the direction of the tilting angle deviation  $(\varphi, \psi)$  is in accordance with the load moment, the load moment capacity of bearing decrease, by contraries, it increases:
- (5) Influence of the tilting angle deviation  $(\varphi, \psi)$  on the load forces  $F_x$  is very small.

## 4.2 Influence of deviation On the Dynamic roperties

The conical AMB dynamic properties can be described by current stiffness matrix and position stiffness matrix, current stiffness matrix is mainly controlled by system controller, bearing position stiffness matrix has close relations to rotor center position. In the following, the influences produced by rotor deviation  $(x, y, z, \varphi, \psi)$  upon to rotor dynamic

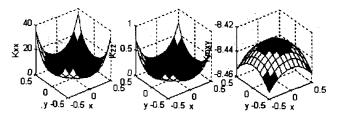


FIGURE 5: Position stiffness coefficient for various deviation x, y

properties are studied.

According Equ (2~7), when rotor at the ideally equilibrium state, namely rotor center is superposition to bearing geometry center, by calculating the dynamic stiffness coefficients, it can be find that bearing position stiffness matrix is a diagonally matrix. When there is a deviation between the rotor centor and the bearing geometry, the position stiffness matrix isn't a diagonally matrix. Fig.5 and Fig.6 show the influences produced by deviation  $(x, y, z, \varphi, \psi)$  on to bearing stiffness coefficient. From these Figures, it can be seen:

(1) In position stiffness matrix, every element is positive, this shows that system position stiffness provides positive feed back, so it make system unstabilization. The parallel deviation x, y of rotor center can enlarge

position stiffness coefficients  $k_{xx}$  and  $k_{yy}$ ;

(2) As shown in Fig.5, when the deviation x, y is smaller, the variety of stiffness coefficient is smaller too; when the deviation is bigger than the variety of stiffness

coefficient is larger.

- (3) In Fig.5, parallel movements (x, y) have a little influences on axial stiffness coefficient  $k_{zz}$ ;
- (4) In Fig.6, the titting angles  $\varphi, \psi$  of the rotor has

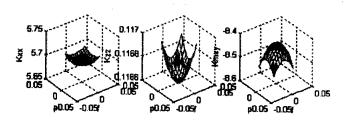


FIGURE 6: position stiffness coefficients for various tilting angles

little influence on force stiffness coefficients, but can affect AMB moment stiffness coefficients badly;

#### 5 CONCLUSIONS

The performances of conical AMB system are mainly influenced by dynamic properties of bearing. The dynamic properties of conical AMB depend on the following three aspects:

- (1) Bearing structure and dimensions.
- (2) Scheme of closed loop controlling system.
- (3) Relative positions between rotor rotating center and bearing geometry center.

This paper first gives the definitions to the dynamic stiffness coefficient matrix for conical bearing, on the basis of which, the varying laws of AMB dynamic properties are studied in the case of non-superposition between rotor rotating center and bearing geometry center. When the rotor is at the equilibrium state, because of some errors in manufacturing or installing some deviation  $(x, y, z, \varphi, \psi)$  in the rotor rotating center relative to the bearing geometric center occurs, which makes the bearing radial clearances are in uneven distribution, so the dynamic properties are affected. Some conclusions are obtained through the calculations and experiments in this paper.

- (1) Parallel deviation movement (x, y) can influence static load capacity and force stiffness coefficients of the system.
- (2) Tilting angles deviation  $(\varphi, \psi)$  can mainly influence static load moment and moment stiffness coefficients.

Since the above variations in stiffness coefficients can block the improvement of rotating speeds and rotating precision, the research contents in this paper will have the great guiding role in designing and analysis conical AMB.

### **REFERENCES:**

- [1]. Michel Dussaux, el., The Industrial Applications of The Active Magnetic Bearings Technology, Proc.2<sup>th</sup> International Symposium on Magnetic Bearings, Japan, Tokyo August 1990
- [2]. Seop Jeong-Ho, el, Robust Motion Control of the Frictionless Positioning Device Suspended by Cone-Shaped Active Magnetic Bearings, Proc. 5<sup>th</sup> International Symposium on Magnetic Bearings, Japan , Kanazawa August 1996
- [3] Jeong, H-S, C-S, Kim, Modeling and Control of Cone-Shaped Active Magnetic Bearings, ETH Zurich, Switzerland, pp, 23-28.1994
- [4] Mohamed, A, and F.Emad, Conical Magnetic Bearings with Radial and Thrust control, IEEE Transactions on Automatic Conteol. Vol. 37, No. 12, pp.1859-1868
- [5]. Fukata, S, Linrarized Model and Control System of Active Magnetic Bearings with Magnet cores in the Shape of a Cone, JSME Series (C), Vol, 58, No, 551, pp75-82, Japan, 1992
- [6].Gen Hai Peng, The Static and Dynamic Characteristics of Conical Magnetic Bearing and Computer Aided Design, Xi'an Jiaotong University, 1991

### APPENDIX:

Nodimension dynamic properties of conical AMB Conmand:

$$\overline{\mu} = \frac{\mu}{\mu_2}, z = h_0 \lambda, \overline{\delta} = \frac{\delta}{\delta_0}, S = \frac{2\pi R h_0}{\cos \beta} \overline{S}, I = I_0 \overline{I}$$

$$\overline{r_h} = \frac{r_h}{h_0}, \overline{N} = \frac{N}{N_0}, N_0 = 200;$$

So nodimension force can be expressed  $\overline{F}, \overline{M}$  :

$$\bar{F}_{xk} = \int_{c_0}^{c_0+h} \int_{\phi_{k1}}^{\phi_{k1}+\phi_0} \bar{f}_1 r t g \beta dz d\phi 
+ \int_{c_0}^{c_0+h} \int_{\phi_{k1}+\phi_0+\phi_m}^{\phi_{k1}+\phi_0+\phi_m} \bar{f}_2 r t g \beta dz d\phi$$

Nodimension coefficient is: 
$$C_0 = \frac{\mu_0 (NIh_0)^2}{2\delta_0^2}$$

### ANNOTATE:

This work was supported by the National Science Foundation of China under Grant 19990511 and National Project 863-2001 AA411310.