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Adaptive Vibration and Unbalance Control of a Rotor Supported by Active Magnetic Bearings

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ABSTRACT

The system under investigation in this paper is a rigid rotor supported by active magnetic bearings (AMB). Two coexisting sources of vibrations which are simultaneously exciting the rotor are considered. First, mass unbalance excitation as a synchronous periodic disturbance and second, destabilizing cross coupling forces. It is shown that any distribution of cross coupling forces along the rotor can be represented by a skew symmetric stiffness matrix with three independent parameters. These parameters may change with time as they depend, for example, on the fluid or gas pressure in the machine and other machine operating conditions.

The basic AMB-control is carried out by a simple PID controller while for the compensation of both excitation mechanisms two independent control algorithms are used. In order to compensate the destabilizing cross coupling forces a standard least square estimator along with a time varying so-called forgetting factor estimates the unknown parameters of the cross coupling mechanism. The mass unbalance forces are compensated by a recursive gain scheduled algorithm which needs no a priori knowledge neither of the amount nor the direction of the mass unbalance. By combining these two algorithms a significant reduction of the vibration level is achieved. The behavior of both algorithms and their interaction with each other is studied by numerical simulation.

INTRODUCTION

The destabilizing effect of nonconservative cross coupling forces is a well known problem in turbomachinery rotordynamics. Usually these forces are described in a linearized form by a skew-symmetric cross coupling stiffness matrix. The source of these forces can be any kind of fluid structure interactions, as, for example, fluid film bearings, annular seals, steam to blade interaction, partially filled centrifuges, or internal damping sources in the shaft.

Usually design measures are taken to ensure the stability in the operating range like swirl breaks, honeycomb-seal stators (see [1]) or squeeze-film dampers. However, reliable experimental data for the cross coupling stiffness coefficients is not available for many of the cross coupling mechanisms and so it may happen that a unit becomes unstable at operating conditions. Another scenario is, that the efficiency or the power of an existing unit should be improved by increasing the operational speed. This will usually lead to increased cross coupled stiffness coefficients and may cause instability.

Active magnetic bearings have become important components in modern turbomachinery. With active bearings it is possible to reduce the vibration level of turbomachinery through various control algorithms. A substantial research has been done on reducing mass unbalance vibrations, especially with so-called adaptive open loop control schemes (see [2]). Probably the most established one of these control methods has been developed by Knospe et al. in [3]. All these algorithms have in common that no a-priori knowledge of neither the amount nor the direction of the mass unbalance is needed.

The control of unstable cross coupling induced vibration is not so widely investigated. In early papers by Matsushita [4] a method is described which uses a certain "tuning filter" to compensate the cross coupling forces. The main drawback of this method is that it needs information on the direction of the whirl as well as the frequency of the unstable vibration. A different approach based on an adaptive pole-assignment controller in conjunction with a state-space model identification is described in [5, 6]. The applied algorithm does not need any a-priori information, but is highly complex and requires high

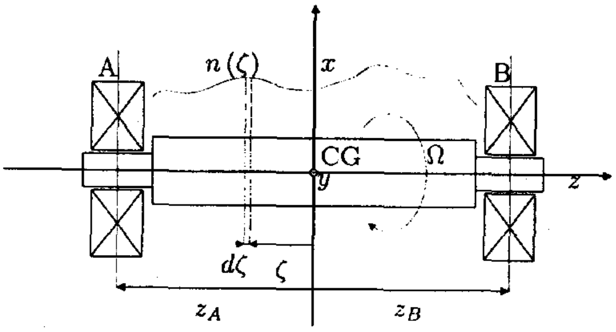

FIGURE 1: Sketch of a rigid rotor supported by active magnetic bearings

computation time.

The first work based on a parameter estimation algorithm was done by Kienberger [7], which was extended and experimentally proven to work by Lang [8, 9]. The location of the cross coupling forces was well-defined and so only one unknown parameter had to be estimated. This algorithm needs no a-priori information about the unstable vibration. Because of the shortness and simplicity of the algorithm it can be easily applied to a bearing-rotor system. It can also be viewed as *an open-loop control method*, since it does not influence the stabilizing underlying control loop. This algorithm is extended in this paper and applied together with an unbalance compensation algorithm.

MODEL OF A RIGID ROTOR SUPPORTED BY ACTIVE MAGNETIC BEARINGS

The system under investigation is illustrated in figure 1 and represents a rigid rotor supported by two active magnetic bearings at stations A and B. An unknown distribution of non-conservative cross coupling forces are acting on the rotor. The cross coupling force density $\frac{d\mathbf{f}_{cc,\zeta}}{d\zeta}$ acting on the rotor at an arbitrary z-position ζ is given by

$$\frac{d\mathbf{f}_{cc,\zeta}}{d\zeta} = -\begin{bmatrix} 0 & n(\zeta) \\ -n(\zeta) & 0 \end{bmatrix} \begin{bmatrix} x(\zeta) \\ y(\zeta) \end{bmatrix} .$$

A positive coefficient $n(\zeta)$ induces a forward whirl and a negative coefficient a backward whirl (for $\Omega > 0$). With a simple linear transformation the force as well as the rotor x and y-position can be transformed to bearing coordinates. With this transformation the cross coupling density can be described as a cross coupling stiffness density matrix

$$\frac{d\mathbf{f}_{cc,b}}{d\zeta} = -n(\zeta) \begin{bmatrix} 0 & v^2 & 0 & -uv \\ -v^2 & 0 & uv & 0 \\ 0 & -uv & 0 & u^2 \\ uv & 0 & -u^2 & 0 \end{bmatrix} \begin{bmatrix} x_A \\ y_A \\ x_B \\ y_B \end{bmatrix}$$

with $u = \frac{(\zeta - z_A)}{(z_A - z_B)}$ and $v = \frac{(\zeta - z_B)}{(z_A - z_B)}$. By integrating this stiffness density matrix over the z-direction of the rotor one gets a skew symmetric cross coupling *stiffness matrix* $\mathbf{N}$ in bearing coordinates with three independent parameters n_1, n_2, n_3

$$\mathbf{f}_{cc,b} = -\mathbf{N}\,\mathbf{x} = -\begin{bmatrix} 0 & n_1 & 0 & n_2 \\ -n_1 & 0 & -n_2 & 0 \\ 0 & n_2 & 0 & n_3 \\ -n_2 & 0 & -n_3 & 0 \end{bmatrix} \begin{bmatrix} x_A \\ y_A \\ x_B \\ y_B \end{bmatrix} . \qquad (1)$$

The Laplace transformed equation of motion of the rotor with constant angular velocity Ω has the form (s is the Laplace variable and a hat above a variable denotes the corresponding Laplace transformed variable)

$$s^2\mathbf{M}\,\hat{\mathbf{x}} + s\,\Omega\mathbf{G}\,\hat{\mathbf{x}} + \mathbf{N}\,\hat{\mathbf{x}} = \frac{1}{sT_{MB}+1}\left(\mathbf{K}_i\,\hat{\mathbf{i}} + \mathbf{K}_s\,\hat{\mathbf{x}}\right) + \hat{\mathbf{d}} \qquad (2)$$

where $\mathbf{M}$ is the mass matrix, $\Omega\mathbf{G}$ is the skew symmetric gyroscopic matrix, $\mathbf{N}$ is the skew symmetric cross coupling stiffness matrix, $\mathbf{K}_s$ is the position coefficient matrix and $\mathbf{K}_i$ is the current coefficient matrix of the two magnetic bearings. $\mathbf{K}_i = k_i\mathbf{I}$ and $\mathbf{K}_s = k_s\mathbf{I}$ are diagonal matrices. $\mathbf{i}(t)$ is the vector of control (reference) currents for the magnetic bearings and $\mathbf{d}(t)$ is the vector of external disturbance forces acting on the rotor. T_{MB} is the time constant of the linearized current controlled active magnetic bearing, which is usually assumed to be zero.

System equ. (2) is unstable due to the negative definite stiffness matrix $-\mathbf{K}_s$ of the magnetic bearings. *In order to stabilize the magnetic levitation* of the rotor a simple PID controller is used. The control law is described as

$$\hat{\mathbf{i}}(s) = -\mathbf{C}(s)\,\hat{\mathbf{x}}(s) + \hat{\mathbf{i}}_n(s) + \hat{\mathbf{i}}_u(s)$$

with $\mathbf{C}(s)$ as the controller Laplace transfer function matrix and $\mathbf{i}_n$, $\mathbf{i}_u$ as additional control currents for cross coupling and unbalance compensation respectively. For $\mathbf{N} = \mathbf{0}$ the controlled system is stable. An increase of one or more of the three parameters n_1 to n_3 will drive the system to instability when exceeding a certain limit. The stability margins of the system for the three parameters are given in figure 2. For any variation of the three parameters the poles are located within two bands as shown in figure 3. The margins of the band are given by the polelocations for a variation of n_2 while n_1 and n_3 is set to zero. The diamonds in figure 3 are indicating the poles of the closed loop system without cross coupling forces. Besides 4 poles close to the origin of the complex plane resulting from the integral part of the controller, there are also two pairs of conjugated complex poles corresponding to a lateral motion and two pairs of conjugated complex poles corresponding to a tilting motion of the rigid rotor.

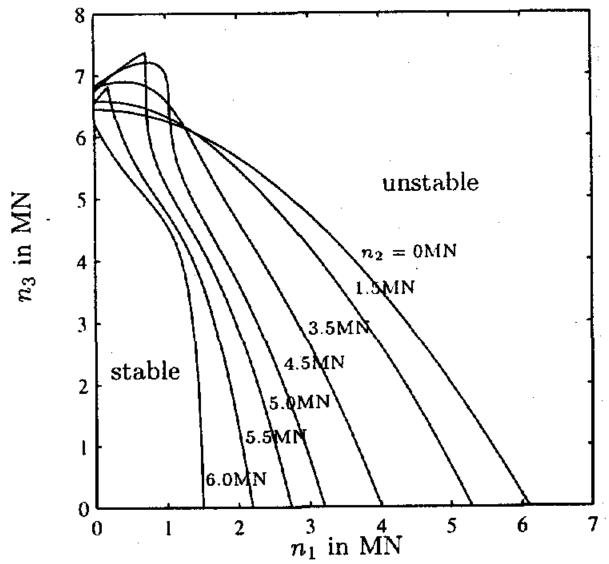

FIGURE 2: Stability margins for cross-coupling parameters n_1 to n_3

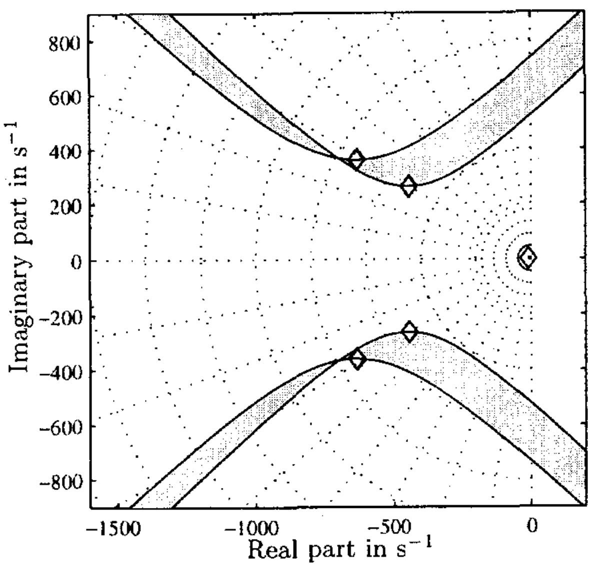

FIGURE 3: Location of the poles of the closed loop system

CROSS COUPLING STIFFNESS ESTIMATION ALGORITHM

In order to stabilize the rotor over a wide range of parameter values an online parameter estimation algorithm is used to estimate the matrix $\mathbf{N}$. With a good approximation $\tilde{\mathbf{N}}$ of $\mathbf{N}$ it is possible to generate counter acting cross coupling forces and stabilize the rotor. The algorithm as used in this paper is presented in [10]. The basic idea is to describe the model of the system used for parameter estimation in a linear parameterized form

$$\mathbf{y}(t) = \mathbf{W}(t)\,\mathbf{n} \tag{3}$$

where the vector $\mathbf{y}$ contains the "outputs" of the system and the vector $\mathbf{n}$ contains the unknown parameters. Note that $\mathbf{y}$ and $\mathbf{W}(t)$ have to be measured or calculated from measurements. The prediction error vector $\mathbf{e}$ is defined as the difference between the real outputs $\mathbf{y}$ and the estimated outputs $\tilde{\mathbf{y}}$, i.e.

$$\mathbf{e}(t) = \tilde{\mathbf{y}}(t) - \mathbf{y}(t) \qquad \text{with} \qquad \tilde{\mathbf{y}}(t) = \mathbf{W}(t)\,\tilde{\mathbf{n}} \tag{4}$$

The algorithm for tracking the time variant parameter vector $\mathbf{n}$ has the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\mathbf{n}}(t) = -\mathbf{P}(t)\,\mathbf{W}^T\mathbf{e}(t)$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{P}(t) = \lambda(t)\,\mathbf{P}(t) - \mathbf{P}(t)\,\mathbf{W}(t)^T\,\mathbf{W}(t)\,\mathbf{P}(t) \tag{5}$$

with a time varying forgetting factor

$$\lambda(t) = \lambda_0\left(1 - \frac{\|\mathbf{P}\|}{K_0}\right). \tag{6}$$

The forgetting factor $\lambda(t)$ is disposing old data quickly for fast changes in the system output (indicated by a small $\|\mathbf{P}\|$) and slowly for little changes in the system output. λ_0 represents the maximum forgetting rate and K_0 the upper bound of $\|\mathbf{P}\|$.

If it were possible to measure the cross coupling forces directly, the linear parameterized form would be

$$\mathbf{f}_{cc} = \mathbf{W}(t)\,\mathbf{n} = \begin{bmatrix} -y_A & -y_B & 0 \\ x_A & x_B & 0 \\ 0 & -y_A & -y_B \\ 0 & x_A & x_B \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} \tag{7}$$

with the unknown parameter vector $\mathbf{n}$ and the 4×3 matrix $\mathbf{W}(t)$. Instead of the actual cross coupling forces the negative calculated active magnetic bearing forces

$$-\hat{\mathbf{f}}_{MB}(s) = -\mathbf{K}_i\left(\mathbf{C}(s)\,\hat{\mathbf{x}}(s) - \hat{\mathbf{i}}_n(s)\right) + \mathbf{K}_s\hat{\mathbf{x}}(s) \tag{8}$$

are used as the reference force (measured outputs) of the estimator model. The additional current for cross coupling compensation $\mathbf{i}_n$ has to be included here, otherwise not the actual cross coupling forces are estimated but the difference between estimated cross coupling forces and cross coupling compensation forces. Although there is no proof that this approach will work for any kind of controller, it is intuitively clear that a controller, designed to attenuate rotor vibrations, is trying to apply forces on the rotor opposing the actual excitation forces.

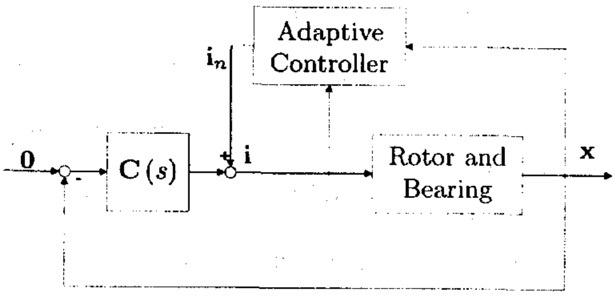

FIGURE 4: Block diagram of the system with adaptive control

It is worth to note that the controller $\mathbf{C}(s)$ used for estimation could also be different from the actual controller, e.g. for a PID controller the integral part of $\mathbf{C}(s)$ could be neglected in equ. (8). However, in this paper the full controller is used and since the controller outputs have to be calculated anyway it is also the least time-consuming method. The resulting additional control current for cross coupling compensation is

$$\mathbf{i}_n = -\mathbf{K}_i^{-1}\tilde{\mathbf{N}}\mathbf{x}\ . \tag{9}$$

The combination of the estimator and cross coupling compensation is referred to in this paper as "Adaptive Controller" (see figure 4). Although the effect of digital control is beyond the scope of this paper it should be noted, that the PID-controller as well as the estimator is simulated as a digital controller with a sample time of 100µs.

NOISE EXCITATION

The most important parameters of the simulation model are given in table 1. Only the linearized parameters of the AMB are given, but the simulation is carried out with a fully nonlinear model.

TABLE 1: Parameters of the simulation model

Parameter	Symbol	Value
Airgap length	l_g	0.5 mm
Rotor mass	m	28.77 kg
Polar mom. of inertia	J_p	0.02188 kgm^2
Current coefficient	k_i	199 N/A
Position coefficient	k_s	1.44 MN/m
Time constant of the current contr. AMB	T_{MB}	14.07 ms

In order to test the sensitivity of the estimation algorithm, band limited white noise with a standard deviation of 0.2µm is added to the measured rotor position signal. The applied cross coupling parameters are increased in steps. The simulation result for the applied values n_1 to n_3 (step functions) and the corresponding estimated values $\tilde{n}_1$ to $\tilde{n}_3$ are shown in the upper graph of figure 5. The lower graph shows the x-displacement of the rotor at bearing station A relative to the airgap length l_g between the stator and the rotor of the magnetic bearing. Every time one or more of the three parameters is changing the system becomes unstable. As an unstable whirl motion of the rotor is developing the estimates of the parameters are getting better and the system is stabilizing again.

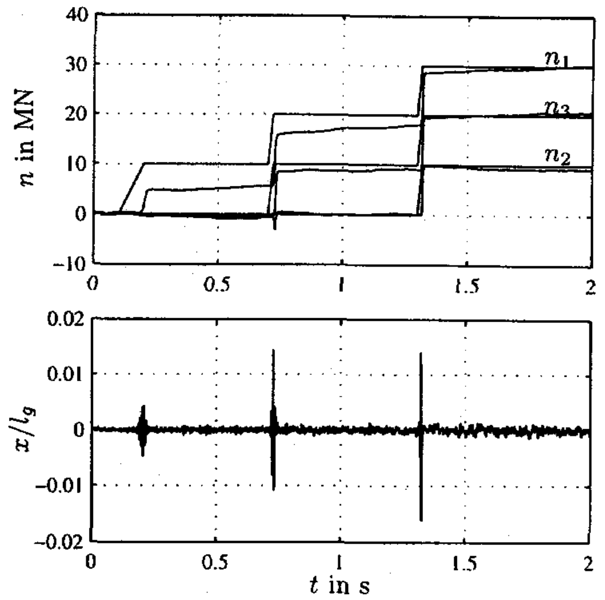

FIGURE 5: Behavior of the adaptive controller in the presence of white measurement noise

HARMONIC EXCITATION

Noise is not the only disturbance in a realistic rotor-bearing system. There are also harmonic disturbances especially speed synchronous first order harmonics due to mass unbalance. To study the effect of harmonic excitation a mass unbalance eccentricity of $e = 2$µm was added and increased to $e = 3$µm by an unbalance jump after 1s. The rotor was run at an operational angular speed $\Omega = 20000$r/min $= 2094$rad s^{-1}. The applied values for the cross coupling parameters are increased from zero to the final values of $n_1 = 18$MN $n_2 = 6$MN and $n_3 = 12$MN. The cross coupling parameters are chosen lower than in the last section because the mass unbalance excitation is pushing the power amplifiers of the AMB to their (rather low) operational voltage limit of 75V. The behavior of the estimator is shown in figure 6. As long as the unstable whirl amplitude is small, the cross coupling forces are small. So the main component in the reference force comes from the harmonic disturbance. The harmonic disturbance is misinterpreted by the estimator as positive cross coupling pa-

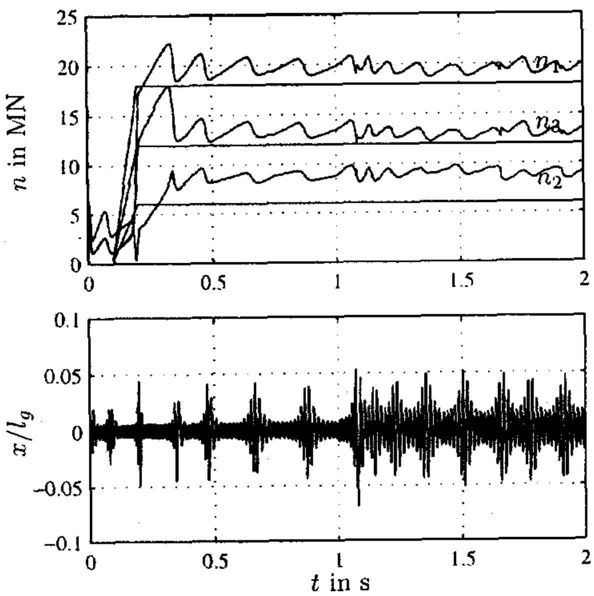

FIGURE 6: Behavior of the adaptive controller in the presence of mass unbalance excitation and measurement noise

rameters and so the estimated parameters $\tilde{n}_1$ to $\tilde{n}_3$ are increasing. As soon as the difference between the applied cross coupling matrix $\mathbf{N}$ and the estimated cross coupling matrix $\tilde{\mathbf{N}}$ is getting high enough an unstable whirl is developing as can be seen from the peaks of the rotor x-displacement in figure 6. During this unstable whirl the cross coupling force is the main component of the reference force and so the estimation tends to the correct value. This mechanism leads to a nonlinear oscillation of the estimated values $\tilde{n}_1$ to $\tilde{n}_3$ as can be clearly seen in the upper graph of figure 6. The frequency of this oscillation is influenced mainly by the maximum forgetting rate λ_0 and the upper bound of $\|\mathbf{P}\|$ of the estimation algorithm.

Due to the "short time instability" the overall vibration level of the rotor is very high (roughly 5% of the airgap length in this example). In order to reduce the effect of harmonic disturbances, filtering of the reference force with a notch filter would lead to the desired results. However, a better way is to compensate the mass unbalance forces directly in the bearing rotor system as shown in the next section of this paper.

REJECTION OF PERIODIC DISTURBANCE FORCES

Various algorithms are available for compensating mass unbalance excited vibrations. In this paper the recursive gain scheduled algorithm as described in [3] is used. It is based on a plant-model of the form

$$\mathbf{x}_1 = \mathbf{T}_1(\Omega)\,\mathbf{i}_1 + \mathbf{x}_{d1} \qquad (10)$$

where $\mathbf{x}_1$ is the $(2l \times 1)$-vector of the first order (speed synchronous) Fourier coefficients of the vibration at l points of interest , $\mathbf{T}_1$ is the $(2l \times 2m)$ influence coefficient matrix, $\mathbf{i}_1$ is the $(2m \times 1)$-vector of Fourier-coefficients of the m control currents and $\mathbf{x}_{d1}$ is the $(2l \times 1)$-vector of Fourier-coefficients of uncontrolled vibrations at l points of interest. This model is only valid for the assumptions that the rotational angular speed Ω is constant, any transients have decayed and only the first order harmonics (denoted by the subscript 1) describe the vibrations. In this form the model is not very useful since it describes a steady state only. By assuming a discrete control schedule which changes the control currents and waits until the steady state is reached the model can be rewritten to

$$\mathbf{x}_{1,k} = \mathbf{T}_1(\Omega)\,\mathbf{i}_{1,k} + \mathbf{x}_{d1,k}. \qquad (11)$$

The subscript k indicates that the quantities are those during the k^{th} schedule. The recursive gain scheduled (RGS) control algorithm consists of iteratively applying the control law

$$\mathbf{i}_{1,k+1} = \mathbf{i}_{1,k} - \mu\mathbf{A}_1\mathbf{x}_{1,k} \qquad (12)$$

with the pseudoinverse $\mathbf{A}_1$

$$\mathbf{A}_1 = \left(\mathbf{T}_1^T\mathbf{T}_1\right)^{-1}\mathbf{T}_1^T \qquad (13)$$

of $\mathbf{T}_1$ and the parameter μ to adjust the rate of convergence. This model assumes that any transient from the last update of the control vector (from schedule $k-1$ to k) have decayed before the Fourier coefficients of the next (schedule k) vibration vector are calculated. This imposes a constraint on the update rate of the control current schedule.

If there are as many points of interest on the rotor as actuators $\mathbf{T}$ is a square matrix. This is the case for the rigid rotor discussed in this paper. Then the pseudoinverse $\mathbf{A}_1$ is equal to the inverse of $\mathbf{T}_1$ and equ. (13) becomes

$$\mathbf{i}_{1,k+1} = \mathbf{i}_{1,k} - \mu\mathbf{T}_1^{-1}\mathbf{x}_{1,k}. \qquad (14)$$

It is worth to note that for the matrix $\mathbf{T}_1$ the inverse is calculated very easily since $\mathbf{K}_i$ is a diagonal matrix.

This control is now added to the PID-controller and cross coupling estimator without changes. To study the effect of the RGS algorithm it is turned on after 0.5 seconds in figure 7. The schedule update time of the RGS algorithm is 0.05 seconds and the parameter is chosen to $\mu = 0.5$. The results are shown in figure 7. After the start of the RGS-control

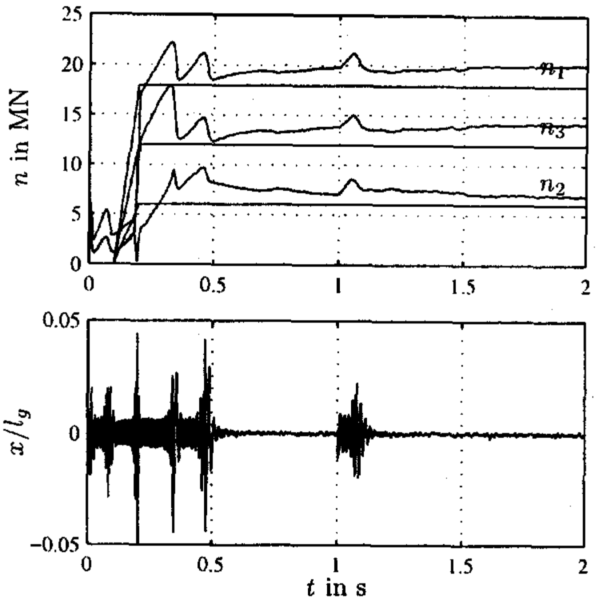

FIGURE 7: Behavior of the adaptive controller in the presence of mass unbalance excitation, measurement noise and RGS-control

(0.5s) the rotor vibrations decay very quickly and the estimated parameters $\tilde{n}_1$ to $\tilde{n}_3$ are hardly changing as the unbalance induced vibrations are reduced. At time 1 second the unbalance suddenly jumps from 2μm to 4μm. Again the estimator is misled by the harmonic vibrations but the RGS-control is able to reduce the vibrations after a few recursions.

SUMMARY

By combining two specific control algorithms a significant reduction of the vibration level of a rigid rotor supported by two magnetic bearings is achieved. The cross coupling compensation algorithm is stabilizing the rotor bearing system that would otherwise be unstable. The recursive gain scheduled algorithm is reducing the vibration level of the system caused by speed synchronous mass unbalance forces and is also improving the estimation process of the first algorithm. This yields a powerful instrument to compensate two of the most dangerous vibration sources in rotating machinery.

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