SYNCHRONOUS UNBALANCE CANCELLATION ACROSS CRITICAL SPEED USING A CLOSED-LOOP METHOD

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ABSTRACT

The subject of our study is to develop a new method that aims at cancelling the effect of the unbalance before crossing the rigid modes, so that the displacement of the rotor corresponds to the run-out from a very low speed through the nominal speed, with a minimal control current.

The general two different approaches in order to achieve unbalance compensation are based on open-loop and closed loop control. Open loop methods don't have stability problems in all speed range, particularly when the rotor goes across the bearing critical speeds. The problems related to open loop compensation are, on the one hand, the initialisation of the compensation algorithm, which can be initiated using an off-line learning method for influence matrix computation, and, on the other hand, an adaptive filter which may introduce spurious disturbances during the speed up of the rotor.

Our purpose is to use a closed loop compensation strategy across the whole critical speed range. This algorithm called AVR (Automatic Vibration Rejection) uses a very simple model in order to schedule automatically the synchronous compensation along the speed range, without any disturbance, even at the first speed up of rotor, with no initialisation.

Because it is a closed-loop method, its influence on the stability of the system needs to be studied.

We use a state-space model extracted from a FEM (finite element method) software in order to get a rough sensitivity transfer function which will be used for the tuning of the AVR parameters.

Because AVR algorithm is implemented on a digital controller, it is now possible to combine the different unbalance cancellation methods available : according to the different applications, we show how it is possible to use vibration cancellation in conjunction with open loop method in order to allow maximum dynamic stiffness strategy, or with the well-known ABS (Active Balancing System) closed loop algorithm, once the rotational speed is far from critical speeds.

This vibration cancellation method is particularly used for serial products such as turbo-molecular pumps for which unbalance cannot be identified for each system. Moreover, thanks to this method, power electronics can be optimised and repetitive balancing operation time can be saved. We finally present experimental results on an air turbine compressor.

INTRODUCTION

In the last few years, the number of rotating machines equipped with Active Magnetic Bearings (AMB) has been remarkably increasing. Owing to its significant advantages, such as contact-less levitation, therefore absence of friction and wear, no need for lubrication and very high rotational speeds [1]. Furthermore, it goes hand in hand with adjustment of the damping, system monitoring and fault detection.

However, AMBs are not as widely spread in industry applications as these advantages might suggest. An important reason for this is the significant complexity of the complete plant in comparison with plants equipped with conventional bearings. It can be pointed out that the turbo-molecular vacuum pump is the most successful application for an AMBs system as a series production. Furthermore, flywheel energy storage systems, general purpose gas blower and other turbo machinery are also interesting application fields for AMBs.

An important problem that concerns every high speed rotating machine will be examined here: the unbalance, responsible for synchronous vibrations. This gap between the axis of inertia and the geometric axis of the rotor is responsible for a great part of the synchronous vibrations of the system. For AMBs system, active regulation can be used as a specific active control to improve unbalance behaviour.

This paper aims at describing a new method for unbalance control that offers particularly good behavioural properties when other compensation methods cannot be used for stability reasons. An accurate model on which our algorithm is tuned will be developed first. We apply this algorithm to a model of a machine in order to validate its limits of stability.

First, the mathematical model including the rigid and the flexible part is given. Then, the problem of unbalance and the methods used to compensate it will are explained. Lastly, the results obtained on an air turbine machine are shown.

1 MODEL OF A ROTATING MACHINE ON AMB

Complex rotors are usually modeled by means of Finite Element Method (FEM) to cope with the non-elementary shape of the shaft and of the rotating appendices connected to it. To account for the complex geometry, the discretization at the base of the FEM model is usually characterized by a high number of nodes and, correspondingly, of degrees of freedom. But modern rotor engineering strongly relies on FE numerical models before undertaking any actual construction. In particular, when AMB are concerned, a model is necessary to design the control law to stabilize the complete system represented by the shaft and the AMBs.

After having built a flexible model for the rotor, we need to take into account the rigid behavioural part of the rotor equipped with AMBs. Then we put together these two halves of the model.

1 – 1 Model of a flexible rotor

A possible output of a FEM software is the frequency response (Bode diagram) of the shaft, which most of the time is enough for the enlarged PID controller design. But this is not enough for time simulations of advanced control algorithms.

What we need is a polyvalent numerical model, not only for designing the control laws needed to stabilize the system but also for simulating anti-vibration algorithms, making time domain simulations and so on.

The first step consists in building a model for the flexible rotor that represent the state of each node. Its size N obviously depends on the number of nodes considered. Let n be the number of nodes, and p the number of degrees of freedom per node. The typical values to define the complete geometry of the rotor are $n=50$ nodes, and $p=4$ degrees of freedom per node. The following equation (1) is the well-known mechanical equation of the flexible system [2].

$$M\ddot{X}+(D+\Omega G)\dot{X}+KX=BF \tag{1}$$

M,D,G,K are respectively the mass, damping, gyroscopic and stiffness matrix. X is the vector containing the np degrees of freedom. F represents the electromagnetic forces, and B the nodes where those forces are applied. Ω is the rotational speed.

We also define the output equation (2), where C corresponds to the nodes we chose to observe.

$$Y=CX \tag{2}$$

The main drawback of the model described by the equations (1) and (2) is its order (usually $N=400$), that makes it uneasy or even heavy to use for calculation. Moreover, such an accuracy concerning all the nodes is not necessary. The behaviour of the system for the first flexible modes (up to 3 or 5 kHz) and for a few number of nodes is sufficient. Only the nodes that are concerned by actuators and measurements import.

A modal reduction of the system with the modal state vectors given by the FEM software is used. The so-called modes are the square roots of the eigenvalues of the $M^{-1}K$ matrix. Let Φ be the matrix composed of the eigenvectors associated to the m flexible modes we want to observe. A new state vector μ of size m as described in equation (3) is used.

$$X=\Phi\mu \tag{3}$$

A matrix of eigenvectors Φ is chosen as to obtain $\Phi^T M\Phi=I$. Let χ be the state vector of the modal state-space model :

$$\chi=\begin{pmatrix}\dot{\mu}\\ \mu\end{pmatrix} \tag{4}$$

The final state-space model for a flexible rotor is defined by the following set of equations (5) and (6) :

$$\dot{\chi}=\begin{pmatrix}-\Phi^T(D+\Omega G)\Phi & -\Phi^T K\Phi\\ I & 0\end{pmatrix}\chi+\begin{pmatrix}\Phi^T B\\ 0\end{pmatrix}F \tag{5}$$

$$Y=\begin{pmatrix}C\Phi & 0\end{pmatrix}\chi \tag{6}$$

The order of this modal state-space model is $2m$. We usually consider the 10 to 15 first flexible modes, so that the resulting modal model has an order 15 to 20 times lower than the one of the original nodal model.

1 – 2 Rigid part of the model

A flexible modal model for the rotor is thus obtained. The rigid part of the system composed of the rotor and the AMBs has to be added in order to complete the model. The rigid behavioural model of the system depends on its geometry, the positions of the actuators and the positions of the detectors [3].

Any movement of a rigid rotor inside its AMBs can be represented as a combination of a translation movement and a tilting movement. Consider the system composed of a bar, two AMBs and the corresponding detectors. In the sequel, the left and right side of the machine will be refered to respectively with "1" and "2" in the subscript notations.

The bearings generate the forces F_1 and F_2. The displacements of the rotor on the detectors are called x_1 and x_2, while x is the displacement of the center of gravity G. α is the angle of rotation of the rotor during a tilting movement around G. L_{b1}, L_{b2}, L_{d1}, L_{d2} are respectively the distances from G to the first and second bearings, and to the first and second detectors. Let J be the axial moment of inertia of the rotor, and M its mass.

The first mechanical equation (7) is related to the movement of G.

$$M\ddot{x} = F_1 + F_2 \tag{7}$$

The second mechanical equation (8) represents the movements of rotation around G for small angles α.

$$J\ddot{\alpha} = F_1 L_{b1} + F_2 L_{b2} \tag{8}$$

With the same hypothesis of small angles, the equations of observation are given by the set (9).

$$\begin{cases} x_1 = x + L_{d1}\alpha \\ x_2 = x + L_{d2}\alpha \end{cases} \tag{9}$$

Equations (7) to (9) can be written under a state-space form, as it appears in equations (10) and (11).

$$\frac{d}{dt}\begin{pmatrix} x \\ \dot{x} \\ \alpha \\ \dot{\alpha} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ \dot{x} \\ \alpha \\ \dot{\alpha} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ M^{-1} & M^{-1} \\ 0 & 0 \\ L_{b1}J^{-1} & L_{b2}J^{-1} \end{pmatrix}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \tag{10}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & L_{d1} & 0 \\ 1 & 0 & L_{d2} & 0 \end{pmatrix}\begin{pmatrix} x \\ \dot{x} \\ \alpha \\ \dot{\alpha} \end{pmatrix} \tag{11}$$

1 – 3 Final model

The final model for the rotor equipped with its magnetic bearings is composed of the sum of two terms : the rigid part of the magnetically suspended body, and the expression of the flexible behaviour of the rotor.

Some other terms of the closed-loop are also taken into account to increase the accuracy of the model. A model for the amplifiers, anti-aliasing filters, detectors, smoothing filters, and of course the numerical controller are then included too. So, this augmented model represents as accurately as possible the behaviour of the system.

2 UNBALANCE CONTROL

2 – 1 Unbalance model

For a rotor, the unbalance represents the difference between the axis of inertia and the geometrical axis imposed by the bearings. This gap comes from the imperfections of the rotor balancing.

During the rotation, the unbalance can be observed by the means of the vibrations it induces on the system. Cancelling the unbalance effect on the system thus consists in merging the rotational axis of the bearings and the axis of inertia of the rotor.

Let m be a mass, placed at a distance d of the axis of inertia of a rotor with a rotational speed of Ω. This mass creates a centrifugal force:

$$F = md\Omega^2 \sin(\Omega t + \varphi) \tag{12}$$

The geometrical unbalance is the md product. If M is the mass of the rotor, this product can be normalized to give:

$$\varepsilon = \frac{md}{M} \tag{13}$$

which is the distance between the axis of inertia and the geometric axis of the rotor.

In order to understand the unbalance compensation method, consider a simplified SISO model of the system, without the gyroscopic effects. $G(j\omega)$ and $K(j\omega)$ respectively stand for the system and the controller.

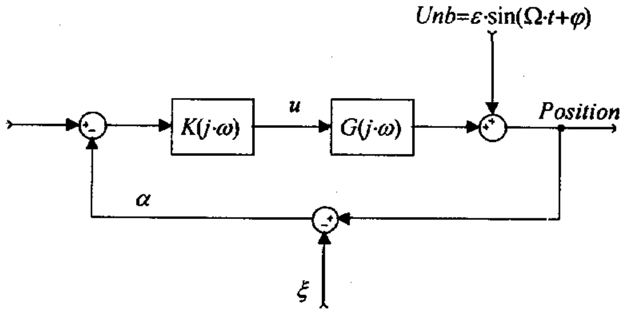

Figure 1: Unbalance model

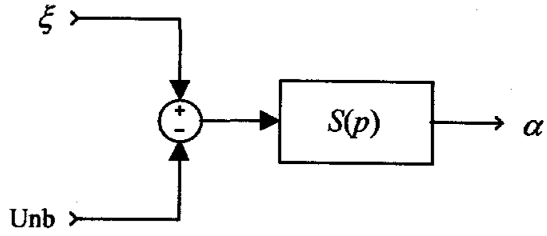

Figure 2 : Diagram equivalent to equation (14)

According to what precedes, the unbalance may be taken into account either as a synchronous disturbance on the force command signal with speed-dependent amplitude and phase, or as a synchronous disturbance on the displacement with a fixed amplitude (geometric synchronous gap). This second description was chosen for a compromise of simplicity. The closed-loop system is shown on figure 1.

$Unb = \varepsilon \sin(\Omega t + \varphi)$ represents the unbalance signal. ξ is an external input for the compensation signal, and α is the measurement used by the control.

2 – 2 Unbalance control

The aim is to minimize the oscillations of the control signal u and conjointly the force due to actuators, responsible for the vibrations, before crossing the rigid mode or around its value. If we choose $\xi = Unb$, the compensation is perfectly achieved and unbalance vibration is no longer present in the control signal u. To reach such a result, *Unb* has to be identified. Actually, an unbalance control algorithm called ABS (Active Balancing System) exists [4], but for stability reasons, it can operate only when the rotational speed is about 20% over the frequency of the rigid mode.

Yet, the objective is to eliminate the vibrations due to unbalance at a speed as low as possible, and above all under the frequency of the rigid mode. That's why a new method that preserve the stability of the system had to be designed.

A new patented method has been developed. It is based on a geometrical interpretation of the system. We consider the following transfer function:

$$\frac{Unb(j\omega)}{\alpha(j\omega)} = \frac{\xi(j\omega)}{\alpha(j\omega)} = \frac{1}{1+G(j\omega)K(j\omega)} = S(j\omega) \qquad (14)$$

This can be expressed under the form of the figure 2, according to the figure 1.

The sinusoidal signals α, ξ and *Unb* can expressed as follows, where Ω is the rotational speed:

$$\alpha = A_\alpha \cos(\Omega t) + B_\alpha \sin(\Omega t) \qquad (15)$$

$$\xi = A_\xi \cos(\Omega t) + B_\xi \sin(\Omega t) \qquad (16)$$

$$Unb = A_{Unb} \cos(\Omega t) + B_{Unb} \sin(\Omega t) \qquad (17)$$

According to those equations, we can set the following figure 3 equivalent to the figure 2. The demodulation necessary to extract A_α and B_α uses the sine and cosine generators, and the low-pass filters, denoted by LPF on the figure.

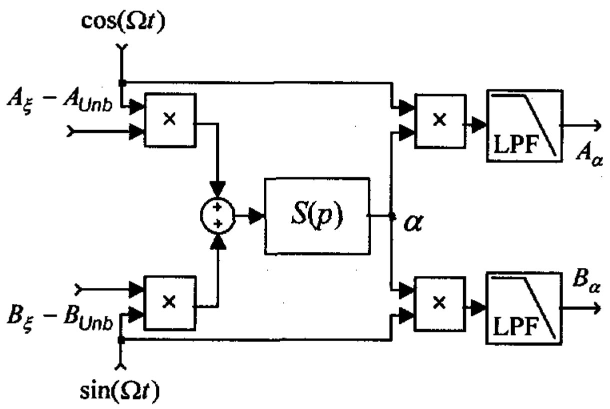

Figure 3 : Equivalent diagram using synchronous parts

The whole figure 3 can be seen along with the compensation feedback loop as described on the figure 4. The matrix M represents a rotation of angle θ, which is the main parameter of the compensation method.

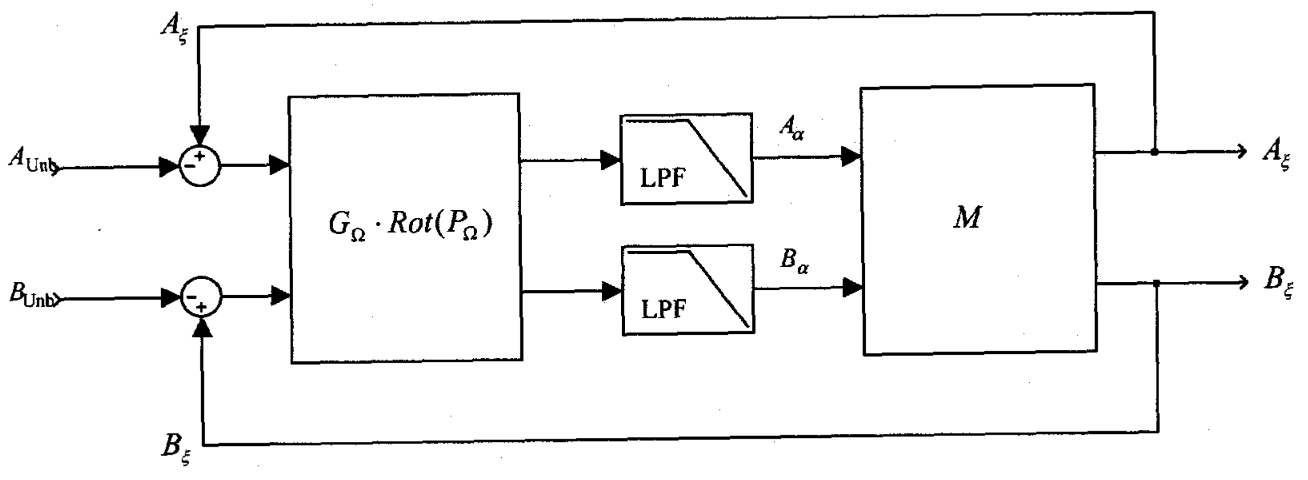

Figure 4 : Principle of the AVR unbalance compensation algorithm

G_Ω and P_Ω are respectively the amplitude and phase of the sensibility function measured at the rotational speed Ω, for the frequency $\omega = \Omega$. The low-pass filter cuts the harmonics further to this operation. The input of the compensation algorithm is the measurement α, and its output is the compensation signal we subtract ξ. We use a high gain for the low-pass filters because the attenuation of the synchronous signal due to unbalance is all the more important since this gain is high.

The angle θ is determined thanks to the phase curve of the sensitivity function. It is meant to compensate the variation of phase of the sensibility function. We use the model in order to predetermine the angle.

The ideal compensation is reached when $A_\xi = A_{Unb}$ and $B_\xi = B_{Unb}$, but thanks to our closed-loop method, a single compensation angle θ keeps the system stable in a large speed range. Using the modeled or measured sensitivity function it is easy to predetermine the stability conditions in a given speed range.

Thanks to numerical control, it is possible to combine different strategies of unbalance control, in order to extend the vibration control in the whole speed range.

These strategies are based on a roughly predetermined varying compensation angle, according to the fact that the phase of the sensitivity function including the compensation angle should not vary more than 90° in the whole speed range. Note that when the compensation angle is zero, the AVR algorithm is equivalent to the well-known ABS algorithm. Generally the vibration control on the whole speed range needs at most 2 or 3 compensation angles, including the nil value (corresponding to ABS) once the rotational speed is far from the critical speeds.

Thanks to the compensation convergence as soon as the rigid body frequencies are reached, it is also possible to freeze the compensation in order to maintain the maximum stiffness, even for synchronous forces, especially in the case of a high speed milling machine (S2M machine SMB30, 30000 rpms, 70 kW), or to cross some other critical speeds.

3 APPLICATION

Results obtained on an air turbine compressor are presented here. In figures 5 and 6, the signals 1, 2, 3 represent respectively the position signal for the right side, the control signal for the left and right side. The figure 5 shows an acceleration for which we use only the classical unbalance compensation method ABS beyond the rigid mode, at 230 Hz.

Due to the geometry of the rotor the right side is less balanced than the left side. Before the ABS is activated, a resonance due to the rigid mode can be noticed. After the ABS is activated, the control signals are minimized, and the position signal has an amplitude equal to the run-out (distance between the axis of inertia and the geometrical axis, see equation 13).

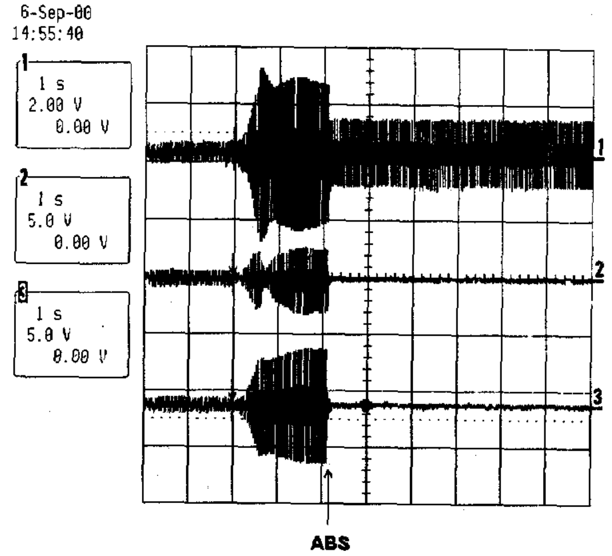

Figure 5 : Acceleration of a pump with ABS

The figure 6 shows an acceleration with the same machine, using the compensation algorithm we developed from 80 Hz to 230 Hz, and the ABS from 230 Hz to the nominal speed. The aim that consisted in minimizing the synchronous control current (thus the synchronous vibrations) is achieved.

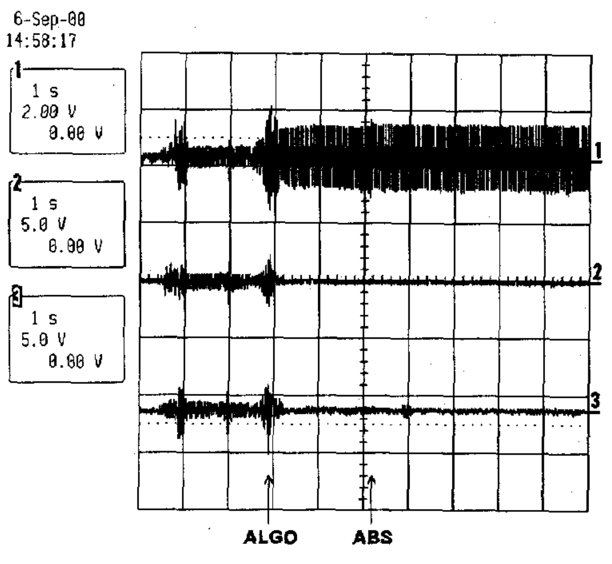

Figure 6 : Acceleration with AVR and ABS

The unbalance compensation method has been also successfully applied to a milling electro-spindle and to a very important industrial application: the turbo-molecular pumps used in the semi-conductor industry. These pumps represent our main series product, and the algorithm developed allows to save time and money on the rotors balancing and the power electronics.

4 CONCLUSION

The objective of this study was to develop an extended method for unbalance compensation of rotating machines equipped with Active Magnetic Bearings (AMBs). This algorithm works for any rotational speed, using a closed-loop method and thus minimizes on the whole range of speed the synchronous vibrations due to unbalance.

A model has been used for the tuning of the unbalance control algorithm. It has been built as a modal state-space model for a flexible and gyroscopic rotor has been extracted from a FEM software.

An algorithm for unbalance suppression before the rigid mode while preserving the stability of the closed-loop system has been developed and successfully applied, and generalized, combined with other unbalance control methods. The tuning of the algorithm relies on the phase curve of the sensibility function, which can be predetermined thanks to the model.

This algorithm has been fruitfully implemented on various applications, particularly on mass-product application; in the case we have showed the robustness of this algorithm without any learning phase of unbalance identification.

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