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UNBALANCE COMPENSATION ON AMB SYSTEM WITHOUT A ROTATIONAL SENSOR

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ABSTRACT

In this paper we propose an unbalance compensation method, which is able to work through a wide range of rotational speed. Actually the vibrations due to unbalance or eccentricity on the rotor contain the direct synchronous rotational speed signal. It is not reasonable using an extra rotational sensor to measure the rotational speed. Thus a resolver-like evaluation of rotational speed is proposed to serve as the necessary synchronous rotational speed signal. The speed of rotation is directly evaluated from the unbalance driven compensation signal.

The unbalance compensation without using a rotational sensor are realized successfully on a test rig with 25 kg-rotor from 0 to 28000 RPM. Moreover, a speed control using the evaluated speed signal is also implemented.

INTRODUCTION

Rotor unbalance is an usual phenomena in rotating machinery, it may induce undesirable vibrations or noise. By using the benefit of contactless levitation in AMB, it is possible to achieve a force-free operation if the rotor rotates about its principal axis.

There are different approaches [1, 2, 3] including feedback and feedforward compensation to achieve such a force free operation, but physically all of the methods lead to a rotation about the principal axis. In order to cancel the synchronous vibration, the rotational speed must be available. It is normally delivered by a tachometer, resolver or another type of rotational sensor. In some applications, it may be difficult or impossible to attach a suitable rotational sensor. If there are electrical machines in the system, the rotational speed can be derived by using self-sensing technique. However, in cases where there are no electrical machines, such methods do not work. In [4] the rotational speed can be derived by adding into a phase-locked loop (PLL)-like adaptation algorithm, which works with internal compensation signal. The converging rate of speed estimation must be set faster than that of compensation adaptation. Obviously a tradeoff exists since increasing the converging rate beyond some limit destabilizes the closed-loop system.

In case of permanent magnet biased magnetic bearings, a current-free operation may be preferred than a force-free operation. In this case, a rotation about the principal axis may cause synchronous bearing forces due to the permanent magnet (PM), if the principal axis is not coincident with the geometrical axis. To ensure a force-free operation, a current must be added to eliminate the bearing forces due to PM. This may lead to saturation of the power amplifier.

In stead of a force-free operation, a current-free operation can be achieved, if the rotor rotates about an axis between its principal axis and geometrical axis (Figure 1). Moreover the control currents are the outputs of the controller, thus they can be accessed directly and precisely without current sensing.

CURRENT-FREE COMPENSATION

In this paper we propose an unbalance compensation method, which is able to work through a wide range of rotational speed. Due to the limited power of the amplifier, we will apply only the current-free operation on the test rig, which is setup with permanent magnet biased magnetic bearings.

In the reality an eccentricity on the sensor ring is almost unavoidable. This causes also a synchronous vibration since the position sensor delivers an error signal. It is theoretically not difficult to eliminate the synchronous vibrations due to rotor unbalance and eccentricity of sensor ring, if the locations of them

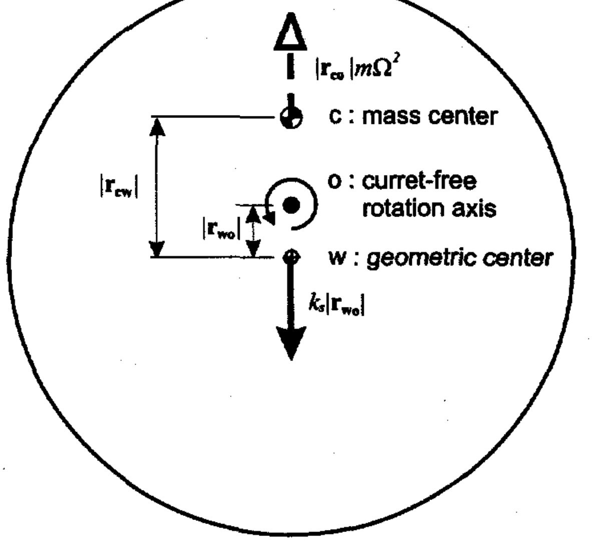

FIGURE 1: current-free operation

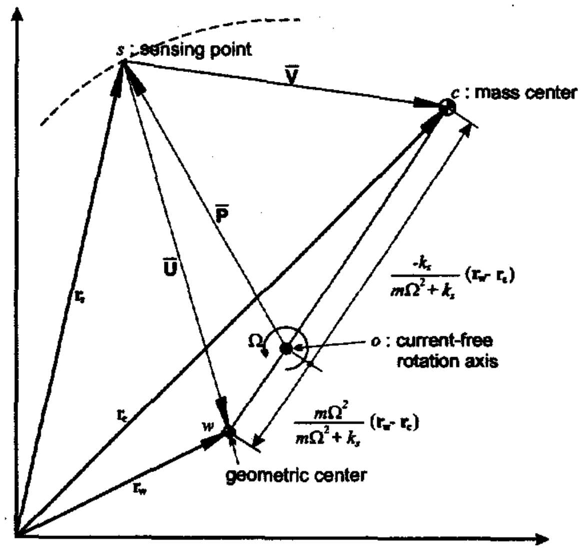

FIGURE 2: current-free compensation of synchronous vibrations

are known. However they are generally unknown amounts and may be changing continuously. It is apparently necessary to have an adaptation algorithm which is able to ensure the current-free operation automatically. Before the derivation we need some definitions; the vectors will be written as complex-numbers:

m	mass
k_s	force-displacement factor
k_{i_m}	current-displacement factor
k_P	gain of P-control
k_D	gain of D-control
f_P	force-factor of P-control, $f_P = k_P k_{i_m}$
f_D	force-factor of D-control, $f_D = k_D k_{i_m}$
$\mathbf{r}_{so} = x_{so} + jy_{so} \in \mathbb{C}$	compensation signal
$\mathbf{i}_\mathbf{m} = i_{m_x} + ji_{m_y} \in \mathbb{C}$	magnetizing current
$\overline{\mathbf{V}}, \overline{\mathbf{U}}, \overline{\mathbf{P}} \in \mathbb{C}$	vectors in rotor-coordinate

In general the bearing force can be given by

$$\mathbf{f}_b = k_s \mathbf{r}_\mathbf{w} - k_{i_m} \mathbf{i}_\mathbf{m} \quad , \tag{1}$$

from the Newton's law we can obtain the equation of motion about mass center c:

$$m\ddot{\mathbf{r}}_\mathbf{c} = \mathbf{f}_b = k_s \mathbf{r}_\mathbf{w} - k_{i_m} \mathbf{i}_\mathbf{m} \tag{2}$$

Now an open-loop synchronous compensation signal $\mathbf{r}_{so}$ should be added to the control loop. The force created by the controlled magnetic bearing is:

$$k_{i_m} \mathbf{i}_\mathbf{m} = f_P(\mathbf{r}_s - \mathbf{r}_{so}) + f_D(\dot{\mathbf{r}}_s - \dot{\mathbf{r}}_{so}) \tag{3}$$

Using the relationships between vectors in Figure 2:

$$\mathbf{r}_{so} = \overline{\mathbf{P}} e^{j\Omega t} \tag{4}$$

$$\mathbf{r}_\mathbf{c} = \mathbf{r}_s + \overline{\mathbf{V}} e^{j\Omega t} \tag{5}$$

$$\mathbf{r}_\mathbf{w} = \mathbf{r}_s + \overline{\mathbf{U}} e^{j\Omega t} \quad , \tag{6}$$

we get the equation of motion about the position sensor's sensing point s:

$$m\ddot{\mathbf{r}}_s = k_s \mathbf{r}_s - k_{i_m} \mathbf{i}_\mathbf{m} + (m\Omega^2 \overline{\mathbf{V}} + k_s \overline{\mathbf{U}}) e^{j\Omega t} \tag{7}$$

Since we want to minimize the magnetizing current $\mathbf{i}_\mathbf{m}$, it is better to write Eq.(7) in form of $\mathbf{i}_\mathbf{m}$:

$$(m\ddot{\mathbf{i}}_\mathbf{m} + c_r \dot{\mathbf{i}}_\mathbf{m} + k_r \mathbf{i}_\mathbf{m}) = \frac{(f_P + jc_r\Omega)}{k_{i_m}} \left[(k_s + m\Omega^2)\overline{\mathbf{P}} + m\Omega^2 \overline{\mathbf{V}} + k_s \overline{\mathbf{U}} \right] e^{j\Omega t} \tag{8}$$

Where, the $c_r = f_D$ is the damping coefficient and $k_r = f_P - k_s$ is the stiffness. It is easy to realize that this equation has the structure of a spring-damper-mass system. The synchronous vibrations deduced by $\overline{\mathbf{U}}$ and $\overline{\mathbf{V}}$ work like a source of disturbance, which drives the vector $\mathbf{i}_\mathbf{m}$.

If we can find a compensation signal $\mathbf{r}_{so} = \overline{\mathbf{P}} e^{j\Omega t}$ which let the right hand side of the Eq.(8) be reduced to zero, then the vector $\mathbf{i}_\mathbf{m}$ will converge to zero too.

In rotor-coordinate we can write:

$$\overline{\mathbf{P}} \longrightarrow -[\frac{1}{\alpha}\overline{\mathbf{V}} + (1 - \frac{1}{\alpha})\overline{\mathbf{U}}] \, , \tag{9}$$

where

$$\alpha = \frac{k_s + m\Omega^2}{m\Omega^2} \quad . \tag{10}$$

Physically the rotor rotates about the point o, which is a point between center of geometry w and center of mass c. Since the position sensor will catch the sensing point s instead of center of geometry w, the compensation signal yields therefore $\overline{\mathbf{P}}e^{j\Omega t}$.

Now we have to find out the vector $\overline{\mathbf{P}}$. Because both of vectors $\overline{\mathbf{U}}$ and $\overline{\mathbf{V}}$ are unknown, it is therefore necessary to have an adaptation algorithm which is able to solve $\overline{\mathbf{P}}$ automatically.

For this purpose Eq.(8) can be rewritten to:

$$m\ddot{\mathbf{i}}_{\mathbf{m}} + c_r\dot{\mathbf{i}}_{\mathbf{m}} + k_r\mathbf{i}_{\mathbf{m}} = \frac{m\Omega^2}{k_{i_m}}(\alpha\overline{\mathbf{R}} + \overline{\mathbf{S}})e^{j\Omega t} , \quad (11)$$

where:

$$\overline{\mathbf{S}} = S_x + jS_y = (f_P + jc_r\Omega)(\overline{\mathbf{V}} + \frac{k_s}{m\Omega^2}\overline{\mathbf{U}}) \quad (12)$$

and

$$\overline{\mathbf{R}} = R_x + jR_y = (f_P + jc_r\Omega)\overline{\mathbf{P}} \quad . \quad (13)$$

Because the right hand side of Eq.(11) is a sine wave excitation, the stationary solution is given by:

$$\mathbf{i}_{\mathbf{m}} = \frac{m\Omega^2}{k_{i_m}} \cdot \frac{(\alpha\overline{\mathbf{R}} + \overline{\mathbf{S}})e^{j\Omega t}}{(k_r - m\Omega^2) + jc_r\Omega} \quad . \quad (14)$$

Now we have to solve an optimization problem: looking for a $\overline{\mathbf{R}}$ which minimizes the energy density of the vector $\mathbf{i}_{\mathbf{m}}$:

$$Q = \frac{1}{2}\left(i_{m_x}^2 + i_{m_y}^2\right) = \frac{1}{2} \cdot \mathbf{i}_{\mathbf{m}} \cdot \mathbf{i}_{\mathbf{m}}^* \quad (15)$$

Using the steepest decent method, the time derivative of $\overline{\mathbf{R}}$ is set in the direction of gradient of the function Q, therefore we can write the adaptation law as following:

$$\begin{aligned} \dot{\overline{\mathbf{R}}} &= -\tau\nabla\mathbf{Q} = -\tau\left(\frac{\partial \mathbf{i}_{\mathbf{m}}}{\partial R_x}\right)^* \mathbf{i}_{\mathbf{m}} \\ &= -\tau\frac{k_s + m\Omega^2}{k_{i_m}}\left[\frac{e^{j\Omega t}}{(k_r - m\Omega^2) + jc_r\Omega}\right]^* \mathbf{i}_{\mathbf{m}} \quad (16) \\ &= -\tau(\gamma_1 + j\gamma_2)e^{-j\Omega t}\mathbf{i}_{\mathbf{m}} \end{aligned}$$

Where τ is the convergency factor, which determines the converging rate of the adaptation. $\frac{1}{\tau}$ corresponds the time constant of a first-order lag. γ_1 and γ_2 are parameters dependent on the rotational speed:

$$\gamma_1 = \frac{k_s + m\Omega^2}{k_{i_m}} \cdot \frac{k_r - m\Omega^2}{(k_r - m\Omega^2)^2 + (c_r\Omega)^2} \quad (17)$$

$$\gamma_2 = \frac{k_s + m\Omega^2}{k_{i_m}} \cdot \frac{c_r\Omega}{(k_r - m\Omega^2)^2 + (c_r\Omega)^2} , \quad (18)$$

Using:

$$\widetilde{\mathbf{R}} = \overline{\mathbf{R}}e^{j\Omega t} \quad (19)$$

the vector $\overline{\mathbf{R}}$ in rotor-coordinate can be transformed into the stator-coordinate $\widetilde{\mathbf{R}}$. Then the adaptive algorithm in stator-coordinate is given by:

$$\dot{\widetilde{\mathbf{R}}} = j\Omega\widetilde{\mathbf{R}} - \tau(\gamma_1 + j\gamma_2)\mathbf{i}_{\mathbf{m}} , \quad (20)$$

or written in matrix-form:

$$\begin{bmatrix} \dot{\widetilde{R_x}} \\ \dot{\widetilde{R_y}} \end{bmatrix} = \mathbf{A}\begin{bmatrix} \widetilde{R_x} \\ \widetilde{R_y} \end{bmatrix} + \mathbf{B}\begin{bmatrix} i_{m_x} \\ i_{m_y} \end{bmatrix} \quad (21)$$

Where

$$\mathbf{A} = \Omega\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad (22)$$

$$\mathbf{B} = -\tau\begin{bmatrix} \gamma_1 & -\gamma_2 \\ \gamma_2 & \gamma_1 \end{bmatrix} . \quad (23)$$

If we look into Eq.(20), we can find that this equation has an integration-like structure. If the magnetizing current $\mathbf{i}_{\mathbf{m}}$ remains nonzero, the vector $\widetilde{\mathbf{R}}$ will be forced to change until $\mathbf{i}_{\mathbf{m}}$ converges to zero. In order to implement the adaptation on a microprocessor, the algorithm must be transformed to discrete form. Setting the sampling period to T_s, we obtain:

$$\begin{bmatrix} \widetilde{R_x}(n+1) \\ \widetilde{R_y}(n+1) \end{bmatrix} = \mathbf{A_d}\begin{bmatrix} \widetilde{R_x}(n) \\ \widetilde{R_y}(n) \end{bmatrix} + \mathbf{B_d}\begin{bmatrix} i_{m_x}(n) \\ i_{m_y}(n) \end{bmatrix} \quad (24)$$

where:

$$\mathbf{A_d} = e^{\mathbf{A}T_s} = \begin{bmatrix} \cos(T_s\Omega) & -\sin(T_s\Omega) \\ \sin(T_s\Omega) & \cos(T_s\Omega) \end{bmatrix} \quad (25)$$

$$\begin{aligned} \mathbf{B_d} &= \mathbf{A}^{-1}\left(e^{\mathbf{A}T_s} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\mathbf{B} \\ &= -\tau \cdot T_s\begin{bmatrix} \xi_1 & -\xi_2 \\ \xi_2 & \xi_1 \end{bmatrix} , \end{aligned} \quad (26)$$

with the parameters:

$$\xi_1 = \frac{1}{T_s\Omega}(\gamma_1\sin(T_s\Omega) - \gamma_2[1 - \cos(T_s\Omega)]) \overset{T_s\Omega\to 0}{\approx} \gamma_1 \quad (27)$$

$$\xi_2 = \frac{1}{T_s\Omega}(\gamma_2\sin(T_s\Omega) + \gamma_1[1 - \cos(T_s\Omega)]) \overset{T_s\Omega\to 0}{\approx} \gamma_2 \quad (28)$$

which approximate to γ_1 and γ_2 while $T_s\Omega$ is very small. With Eq.(4)(13) we obtain the compensation signal:

$$\mathbf{r_{so}} = \frac{\widetilde{\mathbf{R}}}{f_P + jc_r\Omega} \quad (29)$$

and its corresponding matrix-form:

$$\begin{bmatrix} x_{so} \\ y_{so} \end{bmatrix} = \frac{1}{f_P^2 + (c_r\Omega)^2} \begin{bmatrix} f_P & c_r\Omega \\ -c_r\Omega & f_P \end{bmatrix} \begin{bmatrix} \widetilde{R}_x \\ \widetilde{R}_y \end{bmatrix} \tag{30}$$

For the implementation of this adaptation algorithm, we need following parameters:

m, k_s, k_{i_m}: are the known bearing parameters;

f_P, k_r, c_r: are set by controller design;

$\mathbf{i_m}$: *is the control input(output of controller);*

Ω: is the rotational speed, it may be obtained using a tachometer.

Only one of the above parameters: the rotational speed Ω is not always available since in some special applications it is not possible to use a conventional rotational sensor. In order to solve this problem, we will introduce a rotational speed self-sensing technique in the next section.

ESTIMATION OF ROTATIONAL SPEED

In fact the vibrations due to unbalance contain the direct synchronous rotational speed signal, it is therefor not reasonable using an extra rotational sensor to measure the rotational speed. Thus a resolver-like evaluation of rotational speed is proposed to serve as the necessary synchronous rotational speed signal. Not like in [4], the rotational speed is directly evaluated from the unbalance driven compensation signal. Thus it enables a higher dynamics of rotational motion.

It seems to be paradoxical to estimate the speed of rotation from the compensation signal. Since from the adaptation algorithm in the last section we need the speed of rotation at first to set the necessary parameters, then we can obtain the compensation signal. However if we look into Eq.(20), we can see that $\widetilde{\mathbf{R}}$ (and therefore also the compensation signal $\mathbf{r_{so}}$) are driven by $\mathbf{i}_m$. As long as a synchronous disturbance exists (either eccentricity of sensor ring or rotor unbalance), the vector $\mathbf{i_m}$ will rotate with the rotor's rotational speed. Because the system is linear, it is expected that $\mathbf{r_{so}}$ also rotates with the same rotational frequency. Even the magnetizing current $\mathbf{i_m}$ is eliminated by applying compensation, the vector $\mathbf{r_{so}}$ must rotate with the real rotational speed, otherwise $\mathbf{i_m}$ will increase again. So it is clear that the compensation signal $\mathbf{r_{so}}$ always contains the information of rotational speed. It is therefore possible to calculate the speed of rotation from this signal; an eccentricity in system due to manufacturing is from this point of view not only a disturbance, it is also an useful rotational sensor.

It is also clear that this self-sensing technique for the speed of rotation won't work if no eccentricity exists on system. Fortunately (or unfortunately?) it is almost impossible to manufacture a perfect rotor without any eccentricity. There are two possibilities to evaluate the speed of rotation from the compensation signal. Namely:

1. Using the Phase-Locked Loop (PLL), which is wide used in communication techniques, we can measure the frequency of rotation from the x-component x_{so} or from the y-component of $\mathbf{r_{so}}$. However a PLL has its own dynamics, it is therefore not suitable for high dynamic applications.

2. A resolver-like speed calculation will be given here, this enables high dynamic estimation for the rotational speed.

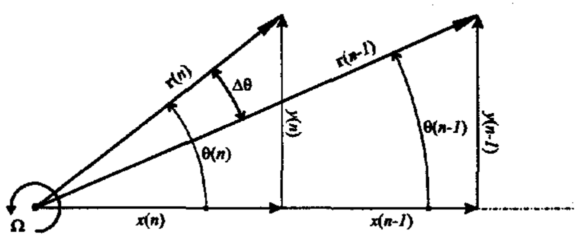

FIGURE 3: rotational speed evaluation

Figure 3 shows the necessary parameters for the corresponding digital implementation. A vector $\mathbf{r}$ is rotating with the speed Ω, the length of the vector doesn't need to be kept constant. From Figure 3 we obtain the angular difference $\Delta\theta$ between 2 sample instants (n) and $(n-1)$:

$$\Delta\theta = \theta(n) - \theta(n-1) = \tan^{-1}\left[\frac{y(n)}{x(n)}\right] - \tan^{-1}\left[\frac{y(n-1)}{x(n-1)}\right] \tag{31}$$

The speed of rotation is given by:

$$\Omega \approx \frac{\Delta\theta}{T_s} = \frac{1}{T_s}\cdot\tan^{-1}\left[\frac{y(n)x(n-1) - x(n)y(n-1)}{x(n)x(n-1) + y(n)y(n-1)}\right] \tag{32}$$

Since this evaluation responds almost immediately to input signals, the estimated speed signal may contain some noise. It is therefore recommended to use a low-pass filter to cancel the high frequency noise before using this speed signal. Figure 4 shows the block diagram of the complete compensation algorithm. The estimated speed signal can not only be used in the unbalance compensation, it can also be used in a sensorless rotational speed control.

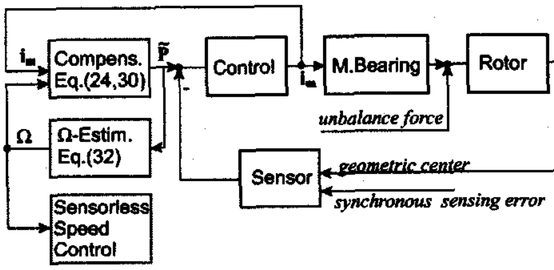

FIGURE 4: complete compensation

SIMULATION

A numerical example is given here for the demonstration of current-free operation combined with self-sensing of rotational speed. The rotor is assumed to be two dimensional with following characteristics:

Rotor:
mass m=10 kg, rotor unbalance with an eccentricity from mass of center $|\mathbf{r_{wc}}| = 0.05$ mm,

bearing parameters:
$k_{i_m} = 100$ N/A, $k_s = 2000$ N/mm,

force-factors:
from P-control $f_P = 3000$ N/mm
from D-control $f_D = 4.47$ N/sec $\cdot$ mm

bearing stiffness:
$k_r = 1000$ N/mm,

damping coefficient:
$c_r = f_D$.

From the above given data we can plot the adaptation parameters γ_1, γ_2 of Eq.(21) and ξ_1, ξ_2 of Eq.(24) in Figure 5. It is apparently that the parameters are not sensible to the variation of rotational speed in the high speed zone. Figure 6 shows the result of simulation. The current-free operation

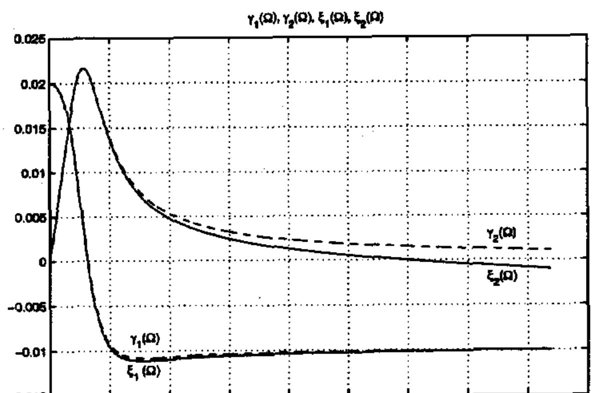

FIGURE 5: Adaptation parameters vs. speed of rotation

is reached within 0.1 sec after the unbalance compensation is enabled. The speed of rotation is also evaluated within a short time (0.05 sec).

It should also be noted that we take the advantage of permanent magnets to suspend the weight of the rotor, thus the rotor will be slight shifted upward so that the magnetic force created by PM just compensates the weight.

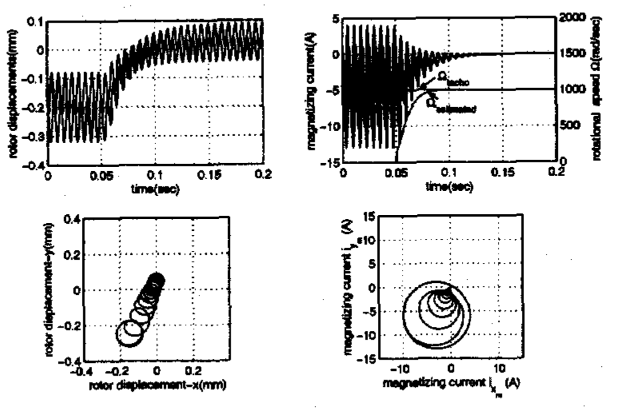

FIGURE 6: Simulation of unbalance compensation

Until now our derivation is based on an ideal 2D-rotor model. In oder to apply this to a real rotor, we can consider a 3D-rotor to be two 2D-rotors, which are suspended separately on two magnetic bearings and their masses are distributed by using the relation $m_1 l_1 = m_2 l_2$ (Figure 7). As long as the angular motions of the rotor keep small, the unbalance compensations can be applied on the planes of bearings separately. This assumption is also validated in the practical experiments.

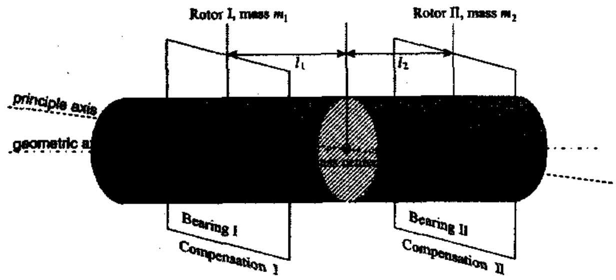

FIGURE 7: separate compensations on two magnetic bearings

EXPERIMENT

Test Rig

The unbalance compensation with rotational speed estimation is validated on a test rig, which is set up with a combined bearing (axial+radial) and a radial bearing. The rotor weighs 25 kg and its length is 670 mm. A 35 kW induction motor is used to drive the rotational motion.

Synchronous Compensation

Figure 8 plots the orbit of sensor-output and current orbit of one bearing as measured by applying the synchronous compensation combined with rotational speed estimation.

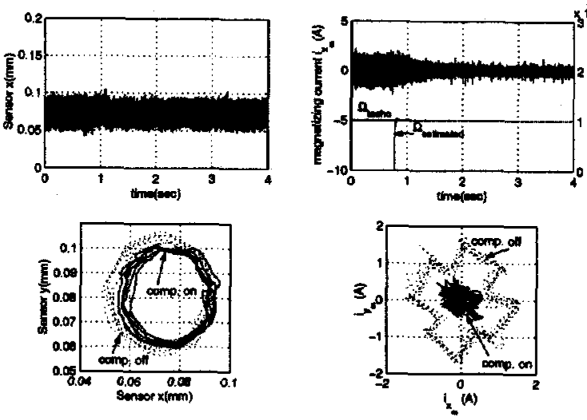

FIGURE 8: Turn on the synchronous compensation at $\Omega = 10000$ U/min

Before the application of compensation the current orbit has unusually the shape of a cross. It may be induced by a manufacturing error on the senor ring, which is slightly more like a triangular than a sinusoidal wave. After the compensation is applied, the current orbit is reduced almost to zero and the rotational speed is evaluated immediately.

Sensorless Speed Control

The estimated speed signal is not only used by the adaptation algorithm, it is also used for the sensorless speed control for the induction motor. Figure 9 shows the experimental results. The motor drives the rotor following a ramp from -10000 RPM to 10000 RPM. It is to be noted that the current orbit is kept almost as a point at the origin during the whole reversal operation. Since the synchronous vibration due to eccentricity on the sensor ring is much more dominating than the rotor unbalance in the low-speed zone, the speed estimation is therefore very much influenced by the bad surface quality of the sensor ring; the speed control works not very well in this zone.

In spite of the manufacturing errors on the rotor, a current-free operation combined with sensorless speed control is implemented through a wide range of operational speed.

CONCLUSION

An adaptation algorithm is developed to compensate synchronous vibrations due to rotor unbalance and

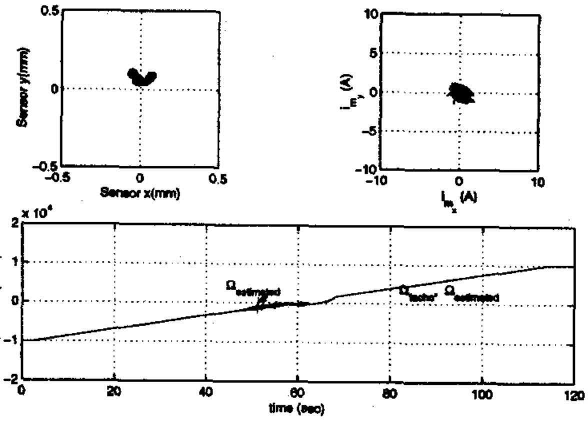

FIGURE 9: Sensorless speed control combined with synchronous compensation

eccentricity on sensor ring. It is combined with a self-sensing mechanism which can detect the speed of rotation. The results have shown that the proposed algorithm can reduce the synchronous vibrations efficiently for a wide range of operating speed. The undesired (and unavoidable) manufacturing eccentricities on rotor is turned into an useful speed detector successfully.

The estimated speed signal is also used to control the rotational speed of the induction motor. However the sensorless speed control in this work is done for an operation with relative slow speed variation. It might be an interesting topic to investigate if this algorithm is also suitable for a high dynamic electrical drive using vector control.

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