8th International Symposium on Magnetic Bearing
August 26-28, 2002, Mito, Japan

Application exact linearization to AMB system
(MIMO exact linerization technique characteristics of a rotor dynamics for AMB system)

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Abstract : In this paper, Input-Output exact linearization techniqu considering modal characteristics of a rotor is proposed for AMB systems. The 1st, the 2nd and the 3rd modes of rotor are takrn into account of control system design. And gyroscopic effects are teken. In order to utilize modal characteristics of the roter with exact Input-Output exact linearization technique, AMB control systems are treated as MIMO systems. The effectiveness of the proposed method is confirmde by levitation and rotating experiments.

1 INTRODUCTION

Nonlinearity of magnetic force is one of difficulties for AMB control. To overcome the difficulty, push-pull coil configuration has been widely used. From the viewpoint control methods, almost all of the AMB control systems are designed as linear due to its simplicity and easiness of the parameter tuning. In general the linear approximation model of the plant around the equilibrium point is adapted for AMB system. But, on the control system designed by using such linear approximation model, the effective range is limited to the points around the equilibrium point. So we can't stably levitate AMB at the distant points from the equilibrium point. In order to improve the performance of AMB control system, it is necessary to develop a nonlinear control method for AMB. On the other hand, owing to the disregard of high order modes, spillover happens and its control system becomes unstable. In this paper, exact linearization technique considering characteristics of a rotor is proposed for AMB systems. As exact linearization technique yields the linear system that is exactly equal to the linear approximation model of the plant around an equilibrium point, we can stably levitate AMB at the points distant from the equilibrium point. In order to utilize modal characteristics of the rotor with exact linearization technique, AMB control systems are treated as MIMO systems. We can control the spillover by taking the high order modes of the rotor into consideration of the control system design. The modal characteristic of the rotor enables us to construct the high performance control systems. But, in case that AMB system are treated as MIMO systems which consider the dynamics of the rotor, the Input-State linearization technique cannot be applicable because of the complexity of models. But, it is shown that the Input-Output exact linearization technique applicable. Based on the results, the nonlinear state feedback controller, i.e. the inputs exchange and the coordinate transformation, is constructed. The MIMO exact linearization of AMB systems enables us to consider the dynamics of the rotor on linear controller design procedure. Firstly the mathematical model of the AMB is introduced and it is confirmed that MIMO exact linearization is applicable to AMB system, and that the inputs exchange and the coordinate transformation and the nonlinear states feedback are derived. The effectiveness of the proposed control method is evaluated by experiments.

2 MIMO MODEL OF AMB

The diagram of a considred AMB rotor model is shown in Fig1.Let the mass of the rotor be M, the left rotor m_1, the second disc m_2, the i-th disc m_i, the right rotor m_n, displacemant from the equilibrium point $x_1, \cdots, x_i$ and x_n respectively. The length from the disc center of gravity to the next disc and the rotor is l.The rotor assumed to be flexible.

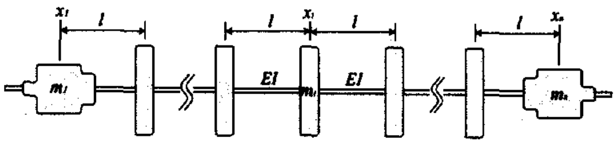

Figure 1: AMB rotor model

$$\boldsymbol{m}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f} \tag{1}$$

where

$$\boldsymbol{m} = \begin{bmatrix} m_1 & & 0 \\ & \ddots & \\ 0 & & m_n \end{bmatrix}, \quad K = [k_{ij}], \quad \boldsymbol{f} = [f_1, \cdots, f_n]^T$$

$$0 \leq i, j \leq n \tag{2}$$

The inductance of upper and lower left coils L_{l1} and L_{l2} are expressed as

$$L_{l1} = \frac{Q}{X + W - x_1} + L_0 \tag{3}$$

$$L_{l2} = \frac{Q}{X + W + x_1} + L_0 \tag{4}$$

where Q,X and L_0 are constant parameters determined by identification experiments, and W is gap between the rotor and bearing at the equiribirium point. Similarly, the inductance of upper and lower right coils L_{r1} and L_{r2} are

$$L_{r1} = \frac{Q}{X + W - x_n} + L_0 \tag{5}$$

$$L_{r2} = \frac{Q}{X + W + x_n} + L_0 \tag{6}$$

The magnrtic levitation forces of left and right AMB are denotd by f_l and f_r respectively. And

$$m_1 = m_n = m_A, \; m_i = m_B \; (i = 2 \sim n - 1) \tag{7}$$

$$f_1 = f_l, \; f_i = 0 \; (i = 2 \sim n - 1), \; f_n = f_r \tag{8}$$

So, the dynamical equations are then expressed as

$$\ddot{x}_1 = -\frac{k_{11}}{m_A}x_1 \cdots - \frac{k_{1j}}{m_A}x_i \cdots - \frac{k_{1n}}{m_A}x_n - \frac{M}{2m_A}g + f_l \tag{9}$$

$$\ddot{x}_i = -\frac{k_{i1}}{m_B}x_1 \cdots - \frac{k_{ii}}{m_B}x_i \cdots - \frac{k_{in}}{m_B}x_n \tag{10}$$

$$\ddot{x}_n = -\frac{k_{n1}}{m_A}x_1 \cdots - \frac{k_{ni}}{m_A}x_i \cdots - \frac{k_{nn}}{m_A}x_n - \frac{M}{2m_A}g + f_r \tag{11}$$

The currents of upper and lower coils of left AMB are i_{l1}, i_{l2}, and i_{r1}, i_{r2}. The magnetic levitation forces f_l and f_r are described as follows:

$$f_l = k\left(\frac{I_1 + \tilde{i}_{l1}}{X + W - x_1}\right)^2 - k\left(\frac{I_2 - \tilde{i}_{l2}}{X + W + x_1}\right)^2 \tag{12}$$

$$f_r = k\left(\frac{I_1 + \tilde{i}_{r1}}{X + W - x_n}\right)^2 - k\left(\frac{I_2 - \tilde{i}_{r2}}{X + W + x_n}\right)^2 \tag{13}$$

where k are constant variables which are determined by identification experiments.

The circuit equations are expressed as

$$\dot{\tilde{i}}_{l1} = -\frac{R\tilde{i}_{l1} + \frac{Q}{(X+W-x_1)^2}\dot{x}_1(I_1 + \tilde{i}_{l1})}{\frac{Q}{X+W-x_1}} + \frac{1}{\frac{Q}{X+W-x_1} + L_0}e_l \tag{14}$$

$$\dot{\tilde{i}}_{l2} = -\frac{R\tilde{i}_{l2} + \frac{Q}{(X+W+x_1)^2}\dot{x}_1(I_2 - \tilde{i}_{l2})}{\frac{Q}{X+W+x_1}} + \frac{1}{\frac{Q}{X+W+x_1} + L_0}e_l \tag{15}$$

$$\dot{\tilde{i}}_{r1} = -\frac{R\tilde{i}_{r1} + \frac{Q}{(X+W-x_n)^2}\dot{x}_n(I_1 + \tilde{i}_{r1})}{\frac{Q}{X+W-x_n}} + \frac{1}{\frac{Q}{X+W-x_n} + L_0}e_r \tag{16}$$

$$\dot{\tilde{i}}_{r2} = -\frac{R\tilde{i}_{r2} + \frac{Q}{(X+W-x_n)^2}\dot{x}_n(I_2 + \tilde{i}_{r2})}{\frac{Q}{X+W-x_n}} + \frac{1}{\frac{Q}{X+W-x_n} + L_0}e_r \tag{17}$$

Now, define the state variables as $\boldsymbol{x} = [x_1 \; \cdots \; x_i \; \cdots \; x_n \; \dot{x}_1 \; \cdots \; \dot{x}_i \; \cdots \; \dot{x}_n \; \tilde{i}_{l1} \; \tilde{i}_{l2} \; \tilde{i}_{r1} \; \tilde{i}_{r2}]^T$.The state space equation then obtained as follows:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_i \\ \vdots \\ \dot{x}_n \\ \hline \alpha_1(x) \\ \vdots \\ \alpha_i(x) \\ \vdots \\ \alpha_n(x) \\ \hline \delta_1(x) \\ \delta_2(x) \\ \delta_3(x) \\ \delta_4(x) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ \hline 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ \hline \frac{1}{\frac{Q}{X+W-x_1}+L_0} & \vdots \\ \frac{1}{\frac{Q}{X+W+x_1}+L_0} & 0 \\ 0 & \frac{1}{\frac{Q}{X+W-x_n}+L_0} \\ 0 & \frac{1}{\frac{Q}{X+W+x_n}+L_0} \end{bmatrix} \begin{bmatrix} e_l \\ e_r \end{bmatrix} \tag{18}$$

$$\alpha_1(x) = -\frac{k_{11}}{m_A}x_1 \cdots - \frac{k_{1j}}{m_A}x_i \cdots - \frac{k_{1n}}{m_A}x_n \tag{19}$$

$$-\frac{3}{2m_A}g + \frac{k}{m_A}\left(\frac{I_1 + \tilde{i}_{l1}}{X + W - x_1}\right)^2 - \frac{k}{m_A}\left(\frac{I_2 - \tilde{i}_{l2}}{X + W + x_1}\right)^2 \tag{20}$$

$$\alpha_i(x) = -\frac{k_{i1}}{m_B}x_1 \cdots - \frac{k_{ii}}{m_B}x_i \cdots - \frac{k_{in}}{m_B}x_n \tag{21}$$

$$\alpha_n(x) = -\frac{k_{n1}}{m_A}x_1 \cdots - \frac{k_{ni}}{m_A}x_i \cdots - \frac{k_{nn}}{m_A}x_n \tag{22}$$

$$-\frac{3}{2m_A}g + \frac{k}{m_A}\left(\frac{I_1 + \tilde{i}_{r1}}{X + W - x_n}\right)^2 - \frac{k}{m_A}\left(\frac{I_2 - \tilde{i}_{r2}}{X + W + x_n}\right)^2 \tag{23}$$

$$\delta_1(x) = -\frac{R\tilde{i}_{l1} + \frac{Q}{(X+W-x_1)^2}\dot{x}_1(I_1 + \tilde{i}_{l1})}{\frac{Q}{X+W-x_1}} \tag{24}$$

$$\delta_2(x) = -\frac{R\tilde{i}_{l2} + \frac{Q}{(X+W+x_1)^2}\dot{x}_1(I_2 - \tilde{i}_{l2})}{\frac{Q}{X+W+x_1}} \tag{25}$$

$$\delta_3(x) = -\frac{R\tilde{i}_{r1} + \frac{Q}{(X+W-x_n)^2}\dot{x}_n(I_1 + \tilde{i}_{r1})}{\frac{Q}{X+W-x_n}} \tag{26}$$

$$\delta_4(x) = -\frac{R\tilde{i}_{r2} + \frac{Q}{(X+W+x_n)^2}\dot{x}_n(I_2 - \tilde{i}_{r2})}{\frac{Q}{X+W+x_n}} \tag{27}$$

3 AMB MODEL

The more natural model is here employed as shown in Fig.2. The control inputs are e for the upper AMB circuit and $-e$ for the lower circuit.

$$e_1 = E_1 + e, \quad e_2 = E_2 - e, \tag{28}$$

where E_1 and E_2 are bias volts for upper and lower AMB circuits. The currents of AMB circuits i_1 and i_2

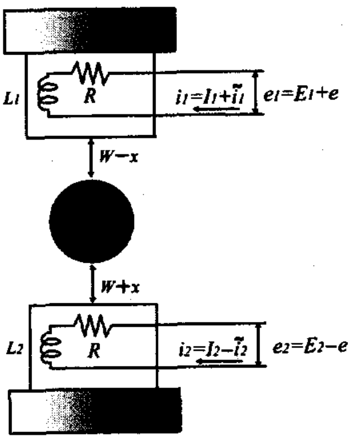

Figure 2: An axis of AMB system (4th order model)

are

$$i_1 = I_1 + \tilde{i}_1, \quad i_2 = I_2 - \tilde{i}_2. \tag{29}$$

where I_1 and I_2 are bias currents caused by bias volts E_1 and E_2. In this case, defining state space variables as $\boldsymbol{x} = \begin{bmatrix} x & \dot{x} & \tilde{i}_1 & \tilde{i}_2 \end{bmatrix}^T$, a state space equation is obtained as follows:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u \\ &= \begin{bmatrix} \dot{x} \\ -g + \frac{Q}{2M}\left(\frac{I_1+\tilde{i}_1}{X_{W-x}}\right)^2 - \frac{Q}{2M}\left(\frac{I_2-\tilde{i}_2}{X_{W+x}}\right)^2 \\ -\frac{R\tilde{i}_1 + \frac{Q}{(X_{W-x})^2}\dot{x}(I_1+\tilde{i}_1)}{\frac{Q}{X_{W-x}}+L_0} \\ -\frac{R\tilde{i}_2 + \frac{Q}{(X_{W+x})^2}\dot{x}(I_2-\tilde{i}_2)}{\frac{Q}{X_{W+x}}+L_0} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\frac{Q}{X_{W-x}}+L_0} \\ \frac{1}{\frac{Q}{X_{W+x}}+L_0} \end{bmatrix} e. \end{aligned} \tag{30}$$

4 MIMO INPUT-OUTPUT LINEARIZATION

In this section, nonlinear control system is derived AMB rotor system. Based on the AMB MIMO model cobsidering rotor modal characteristic, and exact input-output linearization is carried out. First of all, define the output $x_1, \cdots, x_i, \cdots, x_3$, as new state variables $\xi_1, \cdots, \xi_i, \cdots, \xi_3$. Then, also define the derivatives as new state variables and repeat the operation until the input of the system appears in the output as follows

$$x_1 = \xi_1 \tag{31}$$
$$x_i = \xi_i \tag{32}$$
$$x_n = \xi_n \tag{33}$$
$$\dot{\xi}_1 = \dot{x}_1 = \xi_{n+1} \tag{34}$$
$$\dot{\xi}_i = \dot{x}_i = \xi_{n+i} \tag{35}$$
$$\dot{\xi}_n = \dot{x}_n = \xi_{2n} \tag{36}$$
$$\dot{\xi}_{n+1} = \ddot{x}_1 = \alpha_1(x) = \xi_{2n+1} \tag{37}$$
$$\dot{\xi}_{n+i} = \ddot{x}_i = \alpha_i(x) = \xi_{2n+i} \tag{38}$$
$$\dot{\xi}_{2n} = \ddot{x}_n = \alpha_n(x) = \xi_{3n} \tag{39}$$

$$\begin{aligned} \dot{\xi}_{2n+1} &= \dot{\alpha}_1(x) \\ &= -\frac{k_{11}}{m_A}x_1 \cdots - \frac{k_{1i}}{m_A}x_i \cdots - \frac{k_{1n}}{m_A}x_n \\ &\quad + \frac{k}{m_A}\frac{2(I_1+\tilde{i}_{l1})\dot{\tilde{i}}_{l1}}{(X+W-x_1)^2} + \frac{k}{m_A}\frac{2(I_1+\tilde{i}_{l1})^2\dot{x}_1}{(X+W-x_1)^3} \\ &\quad + \frac{k}{m_A}\frac{2(I_2-\tilde{i}_{l2})\dot{\tilde{i}}_{l1}}{(X+W+x_1)^2} + \frac{k}{m_A}\frac{2(I_2-\tilde{i}_{l2})^2\dot{x}_1}{(X+W+x_1)^3} \\ &= -\frac{k_{11}}{m_A}x_1 \cdots - \frac{k_{1i}}{m_A}x_i \cdots - \frac{k_{1n}}{m_A}x_n \\ &\quad + \frac{Q}{m_A}\frac{I_1+\tilde{i}_{l1}}{X+W-x_1}\left(\frac{\dot{\tilde{i}}_{l1}}{X+W-x_1} + \frac{I_1+\tilde{i}_{l1}}{(X+W-x_1)^2}\dot{x}_1\right) \\ &\quad + \frac{Q}{m_A}\frac{I_2-\tilde{i}_{l2}}{X+W+x_1}\left(\frac{\dot{\tilde{i}}_{l2}}{X+W+x_1} + \frac{I_2-\tilde{i}_{l2}}{(X+W+x_1)^2}\dot{x}_1\right) \\ &= -\frac{k_{11}}{m_A}x_1 \cdots - \frac{k_{1i}}{m_A}x_i \cdots - \frac{k_{1n}}{m_A}x_n \\ &\quad + \frac{Q}{m_A}\frac{I_1+\tilde{i}_{l1}}{X+W-x_1}\left(-\frac{R\tilde{i}_{l1} + \frac{Q}{(X+W-x_1)^2}\dot{x}_1(I_1+\tilde{i}_{l1})}{Q+L_0(X+W-x_1)}\right. \\ &\qquad \left. + \frac{1}{Q+L_0(X+W-x_1)}e_l + \frac{I_1+\tilde{i}_{l1}}{(X+W-x_1)^2}\dot{x}_1\right) \\ &\quad + \frac{Q}{m_A}\frac{I_2-\tilde{i}_{l2}}{X+W+x_1}\left(-\frac{R\tilde{i}_{l2} + \frac{Q}{(X+W+x_1)^2}\dot{x}_1(I_2-\tilde{i}_{l2})}{Q+L_0(X+W+x_1)}\right. \\ &\qquad \left. + + \frac{1}{Q+L_0(X+W+x_1)}e_l + \frac{I_2-\tilde{i}_{l2}}{(X+W+x_1)^2}\dot{x}_1\right) \\ &= -\frac{k_{11}}{m_A}x_1 \cdots - \frac{k_{1i}}{m_A}x_i \cdots - \frac{k_{1n}}{m_A}x_n \\ &\quad + \frac{Q}{m_A}\frac{\dot{x}_1(I_1+\tilde{i}_{l1})^2}{(X+W-x_1)^3} - \frac{Q}{m_A}\frac{I_1+\tilde{i}_{l1}}{X+W-x_1}\frac{R\tilde{i}_{l1} + \frac{Q}{(X+W-x_1)^2}\dot{x}_1(I_1+\tilde{i}_{l1})}{Q+L_0(X+W-x_1)} \\ &\quad + \frac{Q}{m_A}\frac{\dot{x}_1(I_2-\tilde{i}_{l2})^2}{(X+W+x_1)^3} - \frac{Q}{m_A}\frac{I_2-\tilde{i}_{l2}}{X+W+x_1}\frac{R\tilde{i}_{l2} + \frac{Q}{(X+W+x_1)^2}\dot{x}_1(I_2-\tilde{i}_{l2})}{Q+L_0(X+W+x_1)} \\ &\quad + \left(\frac{Q}{m_A}\frac{I_1+\tilde{i}_{l1}}{X+W-x_1}\frac{1}{Q+L_0(X+W-x_1)}\right. \\ &\qquad \left. + \frac{Q}{m_A}\frac{I_2-\tilde{i}_{l2}}{X+W+x_1}\frac{1}{Q+L_0(X+W+x_1)}\right)e_l \end{aligned} \tag{40}$$

$$\dot{\xi}_{2n+i} = \dot{\alpha}_i(x) = -\frac{k_{i1}}{m_B}\dot{x}_1 \cdots - \frac{k_{ii}}{m_B}\dot{x}_i \cdots - \frac{k_{in}}{m_B}\dot{x}_n \tag{41}$$

$$\begin{aligned} \dot{\xi}_{3n} &= \dot{\alpha}_n(x) \\ &= -\frac{k_{n1}}{m_A}\dot{x}_1 - \frac{k_{ni}}{m_A}\dot{x}_i - \frac{k_{nn}}{m_A}\dot{x}_n \\ &\quad + \frac{Q}{m_A}\frac{\dot{x}_n(I_1+\tilde{i}_{r1})^2}{(X+W-x_n)^3} - \frac{Q}{m_A}\frac{I_1+\tilde{i}_{r1}}{X+W-x_n}\frac{R\tilde{i}_{r1} + \frac{Q}{(X+W-x_n)^2}\dot{x}_n(I_1+\tilde{i}_{r1})}{Q+L_0(X+W-x_n)} \\ &\quad + \frac{Q}{m_A}\frac{\dot{x}_n(I_2-\tilde{i}_{r2})^2}{(X+W+x_n)^3} - \frac{Q}{m_A}\frac{I_2-\tilde{i}_{r2}}{X+W+x_n}\frac{R\tilde{i}_{r2} + \frac{Q}{(X+W+x_n)^2}\dot{x}_n(I_2-\tilde{i}_{r2})}{Q+L_0(X+W+x_n)} \\ &\quad + \left(\frac{Q}{m_A}\frac{I_1+\tilde{i}_{r1}}{X+W-x_1}\frac{1}{Q+L_0(X+W-x_1)}\right. \\ &\qquad \left. + \frac{Q}{m_A}\frac{I_2-\tilde{i}_{r2}}{X+W+x_n}\frac{1}{Q+L_0(X+W+x_n)}\right)e_r \end{aligned} \tag{42}$$

Now, define the state variables as $\boldsymbol{\xi} = [\xi_1, \xi_2, \cdots, \xi_{3n-1}, \xi_{3n}]^T$. The linearized state space equation then obtained as follow:

$$\frac{d\boldsymbol{\xi}}{dt} = \begin{bmatrix} O & E_n & O \\ O & O & E_n \\ O & K'_n & O \end{bmatrix} \boldsymbol{\xi} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ f_l(x) \\ 0 \\ \vdots \\ 0 \\ f_r(x) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ g_l(x) & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & g_r(x) \end{bmatrix} \begin{bmatrix} e_l \\ e_r \end{bmatrix} \tag{43}$$

$$K'_n = \begin{bmatrix} -\frac{k_{11}}{m_A} & \cdots & -\frac{k_{1i}}{m_A} & \cdots & -\frac{k_{1n}}{m_A} \\ -\frac{k_{21}}{m_B} & \cdots & -\frac{k_{2i}}{m_B} & \cdots & -\frac{k_{2n}}{m_B} \\ \vdots & \ddots & \vdots & & \vdots \\ -\frac{k_{i1}}{m_B} & \cdots & -\frac{k_{ii}}{m_B} & \cdots & -\frac{k_{in}}{m_B} \\ \vdots & & \vdots & \ddots & \vdots \\ -\frac{k_{n-11}}{m_B} & \cdots & -\frac{k_{n-1i}}{m_B} & \cdots & -\frac{k_{n-1n}}{m_B} \\ -\frac{k_{n1}}{m_A} & \cdots & -\frac{k_{ni}}{m_A} & \cdots & -\frac{k_{nn}}{m_A} \end{bmatrix} \tag{44}$$

where

$$\begin{aligned} & f_l(x) \\ = & \frac{Q}{m_A}\frac{\dot{x}_1(I_1+\bar{i}_{l1})^2}{(X+W-x_1)^3} - \frac{Q}{m_A}\frac{I_1+\bar{i}_{l1}}{X+W-x_1}\frac{R\bar{i}_{l1}+\frac{Q}{(X+W-x_1)^2}\dot{x}_1(I_1+\bar{i}_{l1})}{Q+L_0(X+W-x_1)} \\ & + \frac{Q}{m_A}\frac{\dot{x}_1(I_2-\bar{i}_{l2})^2}{(X+W+x_1)^3} - \frac{Q}{m_A}\frac{I_2-\bar{i}_{l2}}{X+W+x_1}\frac{R\bar{i}_{l2}+\frac{Q}{(X+W+x_1)^2}\dot{x}_1(I_2-\bar{i}_{l2})}{Q+L_0(X+W+x_1)} \end{aligned} \tag{45}$$

$$\begin{aligned} & f_r(x) \\ = & \frac{Q}{m_A}\frac{\dot{x}_3(I_1+\bar{i}_{r1})^2}{(X+W-x_3)^3} - \frac{Q}{m_A}\frac{I_1+\bar{i}_{r1}}{X+W-x_3}\frac{R\bar{i}_{r1}+\frac{Q}{(X+W-x_3)^2}\dot{x}_1(I_1+\bar{i}_{r1})}{Q+L_0(X+W-x_3)} \\ & + \frac{Q}{m_A}\frac{\dot{x}_3(I_2-\bar{i}_{r2})^2}{(X+W+x_3)^3} - \frac{Q}{m_A}\frac{I_2-\bar{i}_{r2}}{X+W+x_3}\frac{R\bar{i}_{r2}+\frac{Q}{(X+W+x_3)^2}\dot{x}_3(I_2-\bar{i}_{r2})}{Q+L_0(X+W+x_3)} \end{aligned} \tag{46}$$

$$\begin{aligned} & g_l(x) \\ = & \left(\frac{Q}{m_A}\frac{I_1+\bar{i}_{l1}}{X+W-x_1}\frac{1}{Q+L_0(X+W-x_1)}\right. \\ & \left. + \frac{Q}{m_A}\frac{I_2-\bar{i}_{l2}}{X+W+x_1}\frac{1}{Q+L_0(X+W+x_1)}\right) \end{aligned} \tag{47}$$

$$\begin{aligned} & g_r(x) \\ = & \left(\frac{Q}{m_A}\frac{I_1+\bar{i}_{r1}}{X+W-x_3}\frac{1}{Q+L_0(X+W-x_3)}\right. \\ & \left. + \frac{Q}{m_A}\frac{I_2-\bar{i}_{r2}}{X+W+x_3}\frac{1}{Q+L_0(X+W+x_3)}\right) \end{aligned} \tag{48}$$

Using the input transformation

$$e_l = \frac{1}{g_l(x)}\{v_l - f_l(x)\} \tag{49}$$

$$e_r = \frac{1}{g_r(x)}\{v_r - f_r(x)\} \tag{50}$$

the linearized state equation is

$$\frac{d\boldsymbol{\xi}}{dt} = \begin{bmatrix} 0 & E_n & 0 \\ 0 & 0 & E_n \\ 0 & K'_n & 0 \end{bmatrix} \boldsymbol{\xi} + \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 1 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_l \\ v_r \end{bmatrix} \tag{51}$$

The input transformation for input-output exact linearization which preserve the dynamical characteristics at the equiribirium points is

$$\begin{aligned} \begin{bmatrix} e_l \\ e_r \end{bmatrix} = & \begin{bmatrix} g_l(x) & 0 \\ 0 & g_r(x) \end{bmatrix}^{-1} \left[\begin{bmatrix} a_{11} & a_{12} \cdots & a_{1\,3n-1} & a_{1\,3n} \\ a_{21} & a_{21} \cdots & a_{2\,3n-1} & a_{2\,3n} \end{bmatrix} \boldsymbol{\xi} \right. \\ & \left. + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} v_l \\ v_r \end{bmatrix} - \begin{bmatrix} f_l(x) \\ f_r(x) \end{bmatrix} \right] \end{aligned} \tag{52}$$

If it substitutes and calculates

$$\begin{aligned} & \dot{\xi}_{2n+1} \\ = & a_{11}\xi_1 \cdots + a_{1i}\xi_i \cdots + a_{1n}\xi_n \\ & + \left(a_{1n+1} - \frac{k_{11}}{m_A}\right)\xi_{n+1} \cdots + \left(a_{1n+i} - \frac{k_{1i}}{m_A}\right)\xi_{n+i} \cdots + \left(a_{12n} - \frac{k_{1n}}{m_A}\right)\xi_{2n} \\ & + a_{12n+1}\xi_{2n+1} \cdots + a_{12n+i}\xi_{2n+i} + \cdots + a_{13n}\xi_{3n} + b_{11}v_l + b_{12}v_r \end{aligned} \tag{53}$$

$$\begin{aligned} & \dot{\xi}_{3n} \\ = & a_{21}\xi_1 \cdots + a_{2i}\xi_i \cdots + a_{2n}\xi_n \\ & + \left(a_{2n+1} - \frac{k_{n1}}{m_A}\right)\xi_{n+1} \cdots + \left(a_{2n+i} - \frac{k_{ni}}{m_A}\right)\xi_{n+i} \cdots + \left(a_{22n} - \frac{k_{nn}}{m_A}\right)\xi_{2n} \\ & + a_{22n+1}\xi_{2n+1} \cdots + a_{22n+i}\xi_{2n+i} + \cdots + a_{23n}\xi_{3n} + b_{21}v_l + b_{22}v_r \end{aligned} \tag{54}$$

Laplace conversion is performed

$$\begin{aligned} & \left\{s^3 - a_{12n+1}s^2 + \left(\frac{k_{11}}{m_A} - a_{1n+1}\right)s - a_{11}\right\}\Xi_1(s) \\ & + \left\{-a_{12n+2}s^2 + \left(\frac{k_{12}}{m_A} - a_{1n+2}\right)s - a_{12}\right\}\Xi_2(s) \\ & \cdots + \left\{-a_{12n+i}s^2 + \left(\frac{k_{1i}}{m_A} - a_{1n+i}\right)s - a_{1i}\right\}\Xi_i(s) \\ & \cdots + \left\{-a_{13n}s^2 + \left(\frac{k_{1n}}{m_A} - a_{12n}\right)s - a_{1n}\right\}\Xi_n(s) \\ = & \quad b_{11}V_l(s) + b_{12}V_r(s) \end{aligned} \tag{55}$$

$$\begin{aligned} & \left\{-a_{22n+1}s^2 + \left(\frac{k_{n1}}{m_A} - a_{2n+1}\right)s - a_{21}\right\}\Xi_1(s) \\ & \cdots + \left\{-a_{22n+i}s^2 + \left(\frac{k_{ni}}{m_A} - a_{2n+i}\right)s - a_{2i}\right\}\Xi_i(s) \\ & \cdots + \left\{-a_{23n-1}s^2 + \left(\frac{k_{nn-1}}{m_A} - a_{22n-1}\right)s - a_{2n-1}\right\}\Xi_{n-1}(s) \\ & + \left\{s^3 - a_{23n}s^2 + \left(\frac{k_{nn}}{m_A} - a_{22n}\right)s - a_{2n}\right\}\Xi_n(s) \\ = & \quad b_{21}V_l(s) + b_{22}V_r(s) \end{aligned} \tag{56}$$

Coefficients $a_{11} \sim a_{2n}$ and $b_{11} \sim b_{22}$ can be found, if Eq(9), (11), (14), (15), (16), (17) are primary-approximation-alignment-ized and coefficient comparison of the equations and Eq(52), (53) which carried out Laplace conversion is carried out.

when $n = 3$,

$$
\begin{aligned}
a_{11} &= \tfrac{1}{m_A}(A - \tfrac{3EI}{2l^3})\tfrac{R}{L_c} & a_{21} &= \tfrac{3EI}{m_A l^3}\tfrac{R}{L_c} \\
a_{12} &= \tfrac{3EI}{m_A l^3}\tfrac{R}{L_c} & a_{22} &= -\tfrac{3EI}{2m_A l^3}\tfrac{R}{L_c} \\
a_{13} &= -\tfrac{3EI}{2m_A l^3}\tfrac{R}{L_c} & a_{23} &= \tfrac{1}{m_A}(A - \tfrac{3EI}{2l^3})\tfrac{R}{L_c} \\
a_{14} &= \tfrac{1}{m_A}\left(A - \tfrac{B^2+C^2}{L_c}\right) & a_{24} &= 0 \\
a_{15} &= 0 & a_{25} &= 0 \\
a_{16} &= 0 & a_{26} &= \tfrac{1}{m_A}\left(A - \tfrac{B^2+C^2}{L_c}\right) \\
a_{17} &= -\tfrac{R}{L_c} & a_{27} &= 0 \\
a_{18} &= 0 & a_{28} &= 0 \\
a_{19} &= 0 & a_{29} &= -\tfrac{R}{L_c} \\
b_{11} &= \tfrac{B+C}{m_A L_c} & b_{21} &= 0 \\
b_{12} &= 0 & b_{22} &= \tfrac{B+C}{m_A L_c}
\end{aligned}
$$

$$L_c = \frac{Q}{X+W} + L_0$$

$$A = \frac{Q(I_1^2 + I_2^2)}{(X+W)^3}$$

$$B = \frac{QI_1}{(X+W)^2}$$

$$C = \frac{QI_2}{(X+W)^2}$$

5 EXPERIMENTAl RESUlTS

Fig.3 is the picture of the experimental equipment.

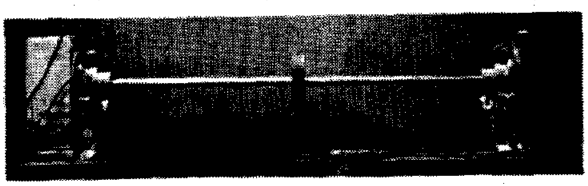

Figure 3: Experimental equipment

Fig.4 is the schematic of AMB system.

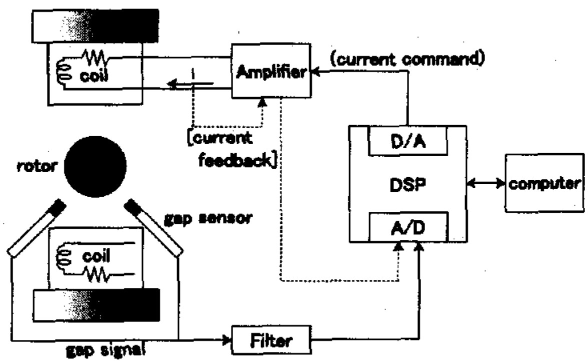

Figure 4: AMB system

5.1 LEVITATION EXPERIMEBNTS

In this section, the results of rotor levitation and rotating experiments with input-output exact linearization technique are shown. The physical parameters of the experimental rig are M: 5.03[kg], m_A: 1.81[kg], m_B: 1.41[kg] g: 9.81[m/s^2], l: 0.355[m], EI: 203[N·m], R: 0.6[Ω], I_1: 2.23[A], I_2: 0.5[A], k: 1.414×10^{-6}, X: 0.1201×10^{-3}[m], W: 0.4×10^{-3}[m], Q: 2.828×10^{-6}[m·H] and L_0: 1.111×10^{-3}[H]. The linear part of the control system is PID controller whose parameters are P: 13000, I: 3000, and D: 30.

Fig.5 shows the experimental result of PID controller with input-output exact linearization. Fig.6 shows the result. The set point ratchets up from the equilibirium point in the experiments. In Fig.6, since the performance of PID controller is poor, the response of the control system begins to oscillate at the point 0.1[mm] distant from the equilibirium point. On the other hand, the control system using nonlinear controller for input-output exact linearization does not however oscillate. By these experimental results, the effectiveness of the input-output exact linearization to enlarge the stability region is confirmed.

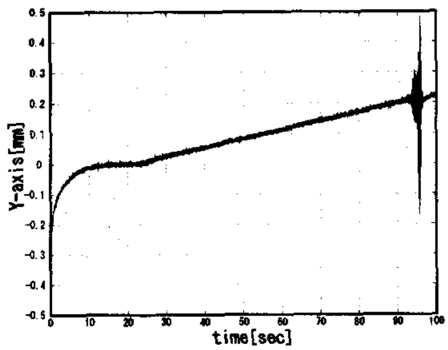

Figure 5: PID with input-output exact linearization

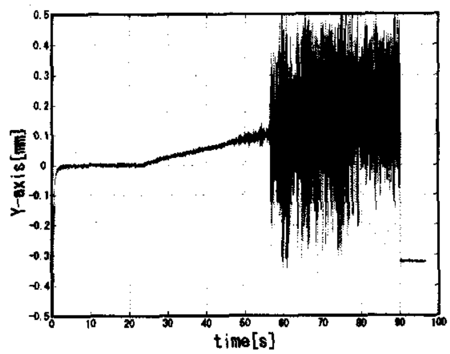

Figure 6: PID controller

5.2 ROTATING EXPERIMENTS

Fig.7, Fig.8, Fig.9 and Fig.10 are results of rotating experiments. Left figures are results of PID controller with input-output exact linearization. Right figures are results of PID controller. The gap of the exigent bearing is 0.2[mm].The linear part of the control system is PID controller whose parameters are P: 8000, I: 3000, and D: 30.

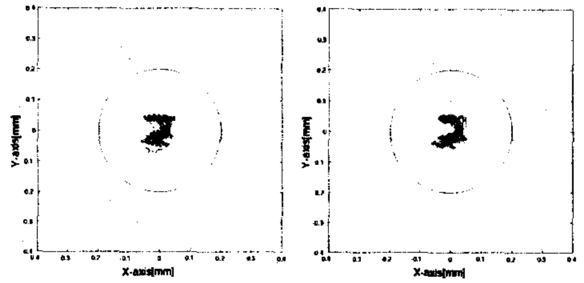

Figure 7: 500[rpm]

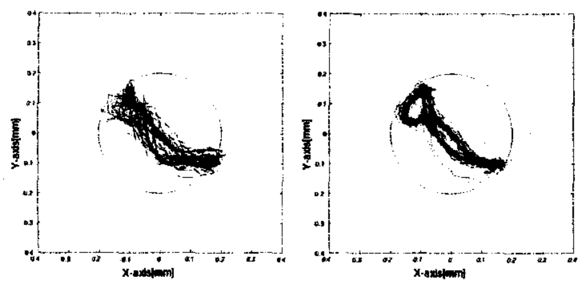

Figure 8: 1200[rpm]

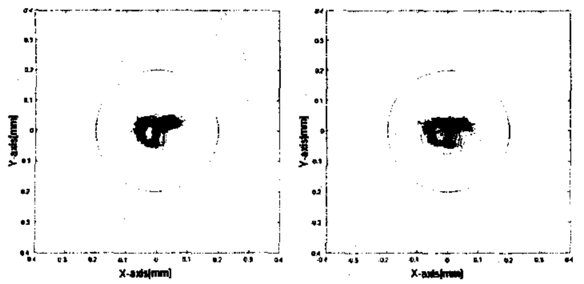

Figure 9: 1700[rpm]

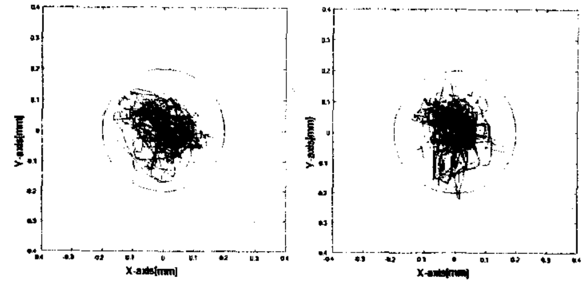

Figure 10: 3500[rpm]

Fig.8 is the diplacements of X-axis and Y-axis at the critical speed of the 1st mode. At this point, both control systems oscillate and get in touch with the exigent bearing. At other speeds of rotation, the control system using nonlinear controller for input-output exact linearization is however more stable than PID controller.

6 CONCLUSION

In this paper, a control system design method for flexible rotor AMB system was proposed. The design method is based on the natural model of the push-pull type AMB system considering rotor modal characteristics. Using input-output exact linearization technique, nonlinear control system was designed. In the case of poor linear controller, it was confirmed that nonlinear controller supported the linear controller and improved the control system performance.

References

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