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POWER-LOSS REDUCTION BY OPTIMIZING CURRENT ALLOCATION IN MAGNETIC BEARINGS

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ABSTRACT

Conventional magnetic bearings control current or flux by operating symmetrically about a bias current or bias flux. This approach is known to be much easier to control than operating without a bias but has the disadvantage of introducing additional power-loss. Although one obvious way to minimize the power-loss is to alternate activation of the two opposing electromagnetic actuators, this single actuator allocation strategy has not been successful in practical applications due to severe performance degradation.

This paper first investigates the fundamental reason behind the performance degradation under the single actuator allocation strategy. One major reason is identified to be voltage saturation in the circuit systems. Based on this result, we then formulate the problem of minimizing the energy consumption by allocating the currents under the constraint of bounded voltages. We establish necessary conditions and properties for the optimal solution, which are used to determine the optimal allocation strategy for some common force signals. Since the optimal solution is very sensitive to the variation of the force signal, we propose a simple static allocation strategy to approximate the optimal solution.

1 INTRODUCTION

Magnetic bearings have several appealing advantages over traditional bearings, such as very low power-loss, very long life, elimination of oil supply, low weight, reduction of oil supply fire hazard, vibration control and diagnostic capability [1]. Even though the power-loss in magnetic bearings is much lower than traditional fluid film bearings, there is still a significant potential for its further reduction. Conventional magnetic bearings control current or flux by operating symmetrically about a bias current or bias flux. This approach is known to be much easier to control than operating without a bias but has the disadvantage of introducing additional power consumption. Much research has been carried out toward reducing the bias or the power consumption (see, e.g., [2, 4, 5, 6, 8]). The obvious way to minimize the power-loss is to activate only one of the pair of electromagnets at a time. This single actuator allocation strategy has not been successful in practical applications due to severe performance degradation. The objective of this paper is to find a way to minimize the power-loss while certain system performances are maintained.

Our study will be based on a simple model which captures the fundamental features of magnetic bearing systems. Consider a rotor whose one dimensional position is controlled by a pair of opposing electromagnets. The dynamics of the rotor can be typically modeled by the following differential equation (see, e.g., [3, 7]):

$$M\ddot{x} = -B\dot{x} + F_2 - F_1 + d, \tag{1}$$

where x is the position (or the angular displacement) and M is the mass (or the moment of mass). F_1 and F_2 are the forces (or torques) produced by the two electromagnets and d is the disturbance resulting from, for example, unbalance or aerodynamic loads. The basic requirement is to stabilize the rotor at the equilibrium point $(x, \dot{x}) = (0, 0)$ with net force $F_n := F_2 - F_1$. The forces F_1 and F_2 are generated by two electromagnetic circuits described by the following differential equations (see, e.g., [7]):

$$L_1\dot{I}_1 = v_1 - I_1\frac{dL_1}{dx}\dot{x} - R_1 I_1, \tag{2}$$

$$L_2\dot{I}_2 = v_2 - I_2\frac{dL_2}{dx}\dot{x} - R_2 I_2, \tag{3}$$

where $L_1 = \frac{c_{\ell 1}}{g_0+x}$ and $L_2 = \frac{c_{\ell 2}}{g_0-x}$. The forces are determined from the air gap fluxes in terms of I_1, I_2 and x as follows,

$$F_1 = c_{f1}\left(\frac{g_0 I_1}{g_0+x}\right)^2, \quad F_2 = c_{f2}\left(\frac{g_0 I_2}{g_0-x}\right)^2.$$

For simplicity, we assume in this paper that $c_{\ell 1} = c_{\ell 2} = c_\ell$, $R_1 = R_2 = R$ and $c_{f1} = c_{f2} = c_f$.

Since the mechanical system (1) is very simple and easy to control if any desired force F_n can be exactly generated (or if approximately generated, the error can be considered as a small disturbance), the system performances depend mainly on the tracking of a desired force signal by the actual force generated by the electromagnets.

Experience shows that the objective of improving the force tracking performance and that of reducing power-loss are conflicting. To reach a good balance between these two objectives, we consider the problem of minimizing power-loss while certain force tracking performance measures are maintained. As a first step, we need to identify the fundamental reason behind the performance degradation when the minimal power-loss allocation strategy is used. We will find out in Section 2 that the main reason is that the single actuator allocation strategy may corrupt the assumption of exact following of a desired current signal by the one that is actually generated. This is in turn caused by the saturation of the voltages. In Section 3, we formulate the problem of minimizing energy consumption under the constraint of voltage saturation and then establish the necessary conditions and some properties of the optimal solution. With these necessary conditions and properties, we can determine the exact optimal solution for some class of force signals. Since the optimal solution at a time instant depends on the future value of the force and is very sensitive to the change of the force signal, we will also propose a simple static allocation strategy in Section 4 which will reduce power-loss to a level very close to the optimal one. Section 5 draws some concluding remarks.

2 AN INVESTIGATION OF PERFORMANCE DEGRADATION

As we have discussed in the introduction, the mechanical system (1) is simple and easy to control, and the performances of the magnetic bearing system rely mainly on the performance of tracking a desired force signal. Hence, in this paper, we will focus on the force tracking performance.

2.1 Force Tracking Performance for a Closed-loop System

In the sequel, we will use F_{nd}, I_{1d} and I_{2d} to denote the desired force and current signals and use F_n, I_1 and I_2 to denote the actual force and currents.

The closed-loop design for the circuit systems (2) – (3) can be divided into two parts:

Part 1. Design a current allocation algorithm,

$$I_{1d} = h_1(F_{nd}, x), \quad I_{2d} = h_2(F_{nd}, x),$$

such that

$$c_f\left(\frac{g_0 I_{2d}}{g_0 - x}\right)^2 - c_f\left(\frac{g_0 I_{1d}}{g_0 + x}\right)^2 = F_{nd}.$$

Part 2. Design control laws for each circuit system,

$$v_1 = g_1(I_1, I_{1d}, x, \dot{x}), \quad v_2 = g_2(I_2, I_{2d}, x, \dot{x}),$$

such that the error between I_1 and I_{1d} and the error between I_2 and I_{2d} are sufficiently small.

As we can expect, the actual force F_n generated by the above designed circuit systems would not be exactly the same as F_{nd}. One of our design objectives is to make the tracking error small by properly choosing the functions h_1, h_2, g_1 and g_2. The other objective is to minimize the power-loss. It appears that these two objectives are independent: the force tracking performance only depends on the current tracking performance, which is determined by g_1 and g_2; and the power-loss depends only on h_1 and h_2. But experience shows that these two objectives are conflicting. This section is dedicated to investigating the fundamental reason that causes the conflict.

2.2 Observations on Performance Degradation

In our study, we use a sinusoidal force signal F_{nd} to demonstrate the performance degradation. For simplicity, we consider the case where $(x, \dot{x}) = (0, 0)$. This is meaningful since it is the case when the system operates in its steady state and we are mainly concerned about the power-loss at steady state. Let $L_0 = c_t/g_0$. Then the circuit systems are

$$L_0\dot{I}_1 = v_1 - RI_1, \quad L_0\dot{I}_2 = v_2 - RI_2. \tag{4}$$

To track the desired current signals I_{1d} and I_{2d}, we can use simple feedback laws $v_1 = g_1(I_1, I_{1d}) = k(I_{1d} - I_1)$ and $v_2 = g_2(I_2, I_{2d}) = k(I_{2d} - I_2)$. Because of the actual bounds on v_1 and v_2, different current allocation strategies will result in different tracking performances when the same force signal F_{nd} is desired. We will compare the following two allocation strategies.

1) *Full bias allocation.* Let F_m be the magnitude of the sinusoidal signal $F_{nd}(t)$, then

$$I_{1d}, I_{2d} = (F_m \mp F_{nd})/(2(c_f F_m)^{\frac{1}{2}}).$$

2) *Single actuator allocation.* If $F_{nd} \geq 0$, then $I_{1d} = 0$ and $I_{2d} = (F_{nd}/c_f)^{\frac{1}{2}}$; If $F_{nd} < 0$, then $I_{2d} = 0$ and $I_{1d} = (-F_{nd}/c_f)^{\frac{1}{2}}$. We also call it alternative allocation or no bias allocation.

Example 1 Consider the circuit system model of a beam balancing test rig[3],

$$L_0\dot{I}_1 = v_1 - RI_1, \quad L_0\dot{I}_2 = v_2 - RI_2,$$

where $L_0 = 4.9060 \times 10^{-4}$H and $R = 0.7\Omega$. The coefficient of force is $c_f = 0.1384N/A^2$ and the desired force signal is $F_{nd}(t) = 2\sin(7000t)N$. The

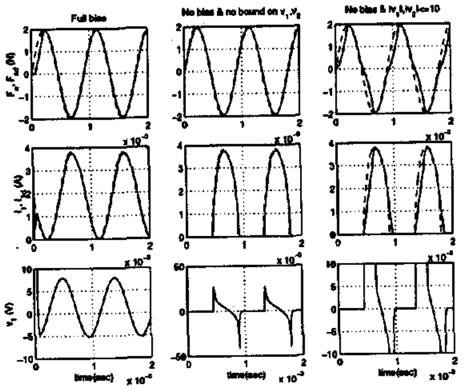

Figure 1: The reason for bad tracking: voltage saturation and improper current allocation

voltage control laws are $v_1 = 50(I_{1d} - I_1)$ and $v_2 = 50(I_{2d} - I_2)$. Fig. 1 illustrates the difference of tracking performances between the two allocation strategies. In Fig. 1, the first row plots the net force F_n (in solid curves) and the desired signal F_{nd} (in dashed curves), the second row plots the current I_1 (in solid curves) and the desired signal I_{1d} (in dashed curves) and the third row plots the voltage v_1. In the first column are the simulation results for the full bias case and the voltages are bounded by $|v_1|, |v_2| \leq 10$. Actually, for this case, the bounds on the voltages does not have an effect since in the steady state, both v_1 and v_2 are within the bound. The second column shows the simulation results for the case of single actuator allocation with no bounds on the voltages. The third column shows the simulation results for the case of single actuator allocation with the voltages bounded by 10V. By comparing the simulation results, we see that if there is no bounds on the voltages, then there is no obvious difference on the tracking of the desired net force between the two allocation strategies. The bad tracking performance in the third column is a result of the combined effects of the voltage bound and the allocation strategy.

3 POWER-LOSS REDUCTION BY OPTIMIZING CURRENT ALLOCATION

The analysis results from last section indicate that voltage saturation is a key factor causing performance degradation. Therefore, in order to achieve the objective of minimizing power-loss while maintaining a desired force tracking performance, we must take into account the bounds on the voltages.

3.1 An Open-loop Optimization Problem

In control theory, many fundamental problems are formulated in terms of open-loop control, such as controllability and time-optimal control. By open-loop control, we mean that the control signal in the whole time interval of interest is determined off-line. Although open-loop control has a lot of disadvantages and is rarely implemented in practical systems, it provides a limit to the performance that can be achieved with any kind of closed-loop control and it also provides us with a guideline for approximating the limit using closed-loop control. In this section, we will first formulate the problem of minimizing power-loss into an open-loop problem. To do this, we need to assume full knowledge of the desired force signal F_{nd} in certain time interval, say, $[0, T]$.

We will consider the case where $(x, \dot{x}) = (0, 0)$. As we have mentioned earlier, this is simple and also meaningful since we are mainly concerned about the power-loss in the steady state. In the open-loop design, all the signals v_1, v_2, I_1 and I_2 are determined off-line such that the desired force F_n is exactly produced. With all the above assumptions, the relation between the force and the currents is,

$$F_n = F_2 - F_1 = c_f(I_2^2 - I_1^2).$$

Given a desired force signal $F_n(t), [0, T]$, our objective is to generate two current signals $I_1(t)$ and $I_2(t)$ to satisfy $F_n(t) = c_f(I_2^2(t) - I_1^2(t))$ and to minimize the total energy during the time interval $[0, T]$,

$$E = c_e \int_0^T (I_2^2 + I_1^2) dt,$$

where c_e is some constant of the magnetic bearing system. The currents are generated by the circuits

$$L_0 \dot{I}_1 = v_1 - RI_1, \quad L_0 \dot{I}_2 = v_2 - RI_2,$$

subject to the bounds on the voltages $|v_1|, |v_2| \leq v_m$. Assume that the initial conditions of the circuit systems, $I_1(0)$ and $I_2(0)$, are free to choose. This is reasonable since T is usually much greater than the transient period and we can take any measure to bring the initial conditions to the desired values (the energy consumption during this transient period can be neglected). The energy minimization problem subject to the constraint of bounded voltages can be formulated as follows,

$$\min_{v_1, v_2, I_1(0), I_2(0)} \int_0^T (I_1^2 + I_2^2) dt, \qquad (5)$$

$$\text{s.t.} \quad a)\ L_0 \dot{I}_1 = v_1 - RI_1, \quad L_0 \dot{I}_2 = v_2 - RI_2,$$
$$c)\ c_f(I_2^2 - I_1^2) = F_n,$$
$$d)\ |v_1| \leq v_m, \quad |v_2| \leq v_m.$$

Here we note that I_1 and I_2 are intermediate variables depending on v_1, v_2, $I_1(0)$ and $I_2(0)$ from a) and b).

3.2 Conditions for the Optimal Solution

To solve problem (5), we introduce five auxiliary scalar functions $\lambda(t), \alpha_1(t), \alpha_2(t), s_1(t)$ and $s_2(t)$, $t \in [0, T]$, and form the Hamiltonian function

$$H = I_1^2 + I_2^2 + \lambda(I_2^2 - I_1^2 - F_n/c_f) + \alpha_1(v_1^2 + s_1^2 - v_m^2) + \alpha_2(v_2^2 + s_2^2 - v_m^2).$$

Using variational method, we obtain the following condition for an optimal solution:

$$\frac{1}{L_0}e^{\frac{Rt}{L_0}}\int_t^T (1-\lambda^*(\tau))I_1^*(\tau)e^{\frac{-R\tau}{L_0}}d\tau+\alpha_1^*(t)v_1^*(t)\equiv 0,$$
$$\frac{1}{L_0}e^{\frac{Rt}{L_0}}\int_t^T (1+\lambda^*(\tau))I_2^*(\tau)e^{\frac{-R\tau}{L_0}}d\tau+\alpha_2^*(t)v_2^*(t)\equiv 0,$$
$$\int_0^T (1-\lambda^*(\tau))I_1^*(\tau)e^{-R\tau/L_0}d\tau\equiv 0,$$
$$\int_0^T (1+\lambda^*(\tau))I_2^*(\tau)e^{-R\tau/L_0}d\tau\equiv 0,$$
$$\alpha_1^*(t)s_1^*(t)\equiv 0,\ \ \alpha_2^*(t)s_2^*(t)\equiv 0,$$
$$(I_2^*(t))^2-(I_1^*(t))^2\equiv F_n(t)/c_f,$$
$$(v_1^*(t))^2+(s_1^*(t))^2\equiv v_m^2,\ \ (v_2^*(t))^2+(s_2^*(t))^2\equiv v_m^2.$$

For simplicity of analysis, we make the following weak assumptions on F_n to exclude some special patterns in F_n.

Assumption 1
1) $F_n(t)$ is continuously differentiable in $(0,T)$ and not identically zero in any sub-interval of $[0,T]$.
2) Let $I_a=(|F_n|/c_f)^{\frac{1}{2}}$. There is no interval $[t_1,t_2]\subset[0,T]$ such that for all $t\in[t_1,t_2]$,

$$RI_a(t)+L_0\dot{I}_a(t)=v_m\ (\text{or}\ -v_m). \quad (6)$$

3) There is no interval $[t_1,t_2]\subset[0,T]$, where there exist I_1 and I_2 such that for all $t\in[t_1,t_2]$,

$$RI_1(t)+L_0\dot{I}_1(t)=\pm v_m,\ \ RI_2(t)+L_0\dot{I}_2(t)=\pm v_m. \quad (7)$$

Without loss of generality, we also assume that $I_1(t),I_2(t)\geq 0$ for all t. Otherwise, we can reverse the sign of the voltage when $I_1(t)$ or $I_2(t)$ starts to decrease from 0.

Based on the stationary condition and since all the variables are piecewise continuous, the interval $[0,T]$ can be divided into sub-intervals (t_i,t_{i+1}), where all the optimal variables are continuous, of the following four types:

Type	I_1^*	I_2^*	$\lvert v_1^*\rvert$	$\lvert v_2^*\rvert$	λ^*
Type A	0	$\neq 0$	$< v_m$	$< v_m$	-1
Type B	$\neq 0$	0	$< v_m$	$< v_m$	1
Type C	$\neq 0$	$\neq 0$	v_m	$< v_m$	-1
Type D	$\neq 0$	$\neq 0$	$< v_m$	v_m	1

The above table means that if both $|v_1|$ and $|v_2|$ are strictly less than v_m, then one of the currents must be zero. On the other hand, if both of the currents are nonzero, then one of the voltages must take the maximum or the minimum value.

For a general force signal F_n, the optimal allocated currents and the corresponding voltage signals usually contain different types of sub-intervals. Although the signals are easy to determine for each given type, it is not a simple task to find out how many sub-intervals are contained and to determine exactly the time instants where one type is switched to another type. Further information about the optimal signals can be revealed after more detailed examination.

Theorem 1 *Let $(I_1^*,I_2^*,v_1^*,v_2^*)$ be a set of optimal signals. Suppose that the first interval is of Type A ($I_1^*=0,I_2^*>0$). The signals can switch to Type B ($I_1^*>0,I_2^*=0$) at $t=t_1$ only if $F_n(t_1)=0$. They can switch to Type D ($|v_1^*|<v_m,v_2^*=\pm v_m$) at $t=t_1$ only if $v_2^*(t_1^-)=\pm v_m$. Consider the case that they switch to Type C (with $v_1^*=v_m,|v_2^*|<v_m$) at $t=t_1$. Assume that $|v_2^*(t_1)|<v_m$. Then the signals will not switch to*

1) Type D before $|v_2^*|$ reaches v_m or before I_2^* reaches 0.

2) Type C (with $v_1^*=-v_m,|v_2^*|<v_m$) before $|v_2^*|$ reaches v_m or before I_2^* reaches 0.

3) Type B before I_2^* reaches 0.

Remark 1 An optimal solution is expected to have Type A or Type B intervals. If $F_n(t)$ is periodic, then it does not matter when we start the signal and it is without loss of generality to assume that the optimal signals start with Type A interval.

Similar result exists if the first interval is of Type B. From Theorem 1, we conclude that, after the signals have switched from Type A to Type C (or from Type B to Type D), there will be no more switch before $|v_1|=|v_2|=v_m$ or before one of the currents reaches 0.

3.3 The Optimal Solution: an Example

In this section, we use an example to illustrate how to obtain the optimal solution using the stationary condition and Theorem 1.

Example 2 Consider the same circuit systems as in Example 1. Suppose that $F_n(t)=2\sin(7000t+\pi/2)$. Since the signal is periodic and symmetric, we only need to consider a half period. Here we have $T=\pi/7000=4.4880\times 10^{-4}$. Let us first plot the optimal signals without considering the voltage constraint in Fig. 2. The actual v_1 and v_2 are unbounded but we clipped off from above $v=20$ and below $v=-20$. Between the interval $[t_a,t_b]$, either v_1 or v_2 exceeds the constraint.

It is clear that Type A or Type B intervals (either $I_1\equiv 0$ or $I_2\equiv 0$) must be outside of $[t_a,t_b]$ and Type A ($I_1\equiv 0$) can only occur between 0 and t_a and Type B ($I_2\equiv 0$) can only occur between t_b and T. We assume that the first interval is Type A.

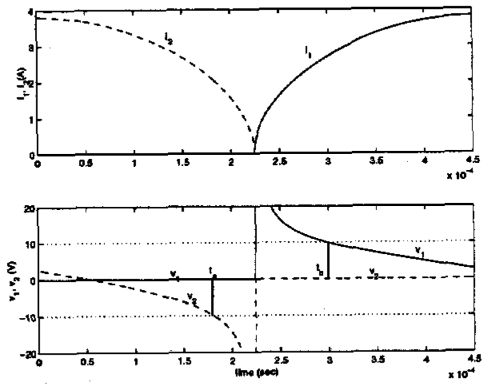

Figure 2: The optimal signals without considering the voltage constraint

It is also obvious that the second interval cannot be of Type B since $[t_a, t_b]$ must be covered with Type C or Type D intervals. We first try the second interval with Type D ($v_2 = \pm v_m$, $|v_1| < v_m$). By Theorem 1, the switching time must be at $t_1 = t_a$ when v_2 reaches $-10V$. Computation reveals that this switch to Type D will result in $v_1 > 10$ for $t > t_1$. Hence we can exclude this possibility and try to switch to Type C at certain t_1. By Theorem 1, there will be no switch to other types after t_1 and before $|v_2| = v_m$ or $I_2 = 0$. With different t_1, we obtained three outcomes:

1) $t_1 > 1.5313$. Infeasible (see Fig. 3, the upper small figure).

2) $t_1 < 1.5313$. Infeasible (see Fig. 3, the lower small figure).

3) $t_1 = 1.5313$. The signal v_2 will not reach $-v_m$ before I_2 reaches 0 at some t_2. The switch to Type B at t_2 results in feasible solution and Type B can be continued until $t = T$ (see Fig. 4).

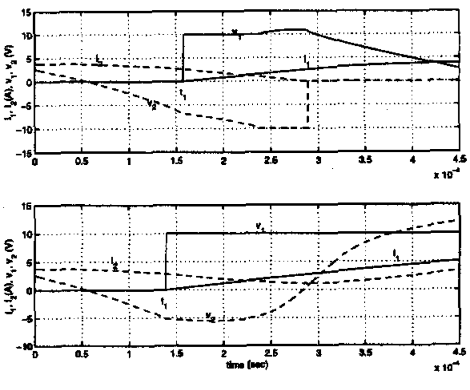

Figure 3: Some non-feasible solutions resulting from different t_1

We see that the third case is the only solution, among other possible optimal solutions, that is feasible for the interval $[0, t_2^+)$. After t_2, switching to any other type will increase the energy consumption. Hence it is the optimal solution. Fig. 4 plots the optimal currents and voltages, where the dashed curves in the current figure are the optimal currents for the case of no bound on the voltages. The minimal energy over the half period $[0, T]$ is found to be $E = 4.3175c_e \times 10^{-3}$.

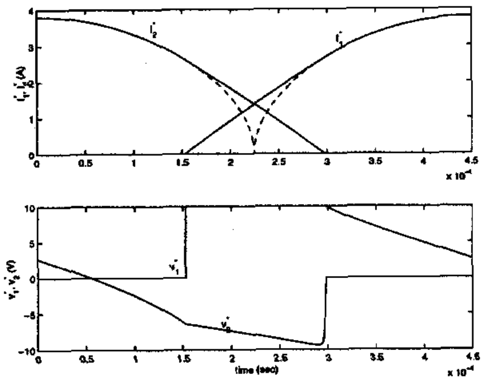

Figure 4: The optimal allocation and the signals

It can be expected that the optimal signals for all the sinusoidal force signals $F_n(t)$ will have the same pattern as demonstrated in Example 2, which completely depends on the first switching time t_1. It turns out that the signals are very sensitive to t_1. A small variation of t_1 from the optimal value will cause nonfeasibility, not just non-optimality, of the resulting signals. Because of this high sensitivity of the optimal solution, we would like to find some static relations $F_n \mapsto I_1$ and $F_n \mapsto I_2$ so that the energy consumption is close to the minimal value achieved by the open loop design.

4 STATIC ALLOCATION STRATEGIES

In this section, we will consider static allocation. We will again use I_{1d}, I_{2d} and F_{nd} to denote the desired current and force signals. By static allocation, we mean that the values of $I_{1d}(t)$ and $I_{2d}(t)$ only depend on $F_{nd}(t)$ for any t. We consider the following class of simple allocation functions with a design parameter F_0,

$$I_{1d} = \begin{cases} 0, & \text{if } F_{nd} > F_0, \\ \frac{F_0 - F_{nd}}{2(F_0 c_f)^{\frac{1}{2}}}, & \text{if } |F_{nd}| \le F_0, \\ (-F_{nd}/c_f)^{\frac{1}{2}}, & \text{if } F_{nd} < -F_0, \end{cases} \qquad (8)$$

and

$$I_{2d} = \begin{cases} 0, & \text{if } F_{nd} < -F_0, \\ \frac{F_0 + F_{nd}}{2(F_0 c_f)^{\frac{1}{2}}}, & \text{if } |F_{nd}| \le F_0, \\ (F_{nd}/c_f)^{\frac{1}{2}}, & \text{if } F_{nd} > F_0. \end{cases} \qquad (9)$$

Fig. 5 plots some functions from this class with $c_f = 0.1384$ and $F_0 = 0, 0.5, 1, 1.5$ and 2 respectively. For

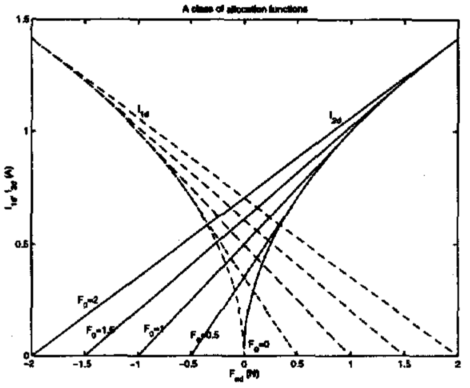

Figure 5: A class of allocation functions

$|F_{nd}| \leq F_0$, the currents are affine in F_{nd}, and for $|F_{nd}| > F_0$, the currents are square roots of $|F_{nd}|/c_f$ or 0. It is clear that smaller F_0 will result in smaller power-loss. The following example will show that a suitable static allocation can reduce the power-loss to a level very close to that by the optimal non-static allocation.

Example 3 We consider the same circuit systems and force signal as in Example 2, $F_{nd}(t) = 2\sin(7000t + \pi/2)$. We use static allocation functions from the class in (8) and (9) to determine $I_{1d}(t)$ and $I_{2d}(t)$. The voltages are determined from

$$v_1 = L_0 \dot{I}_{1d} + R I_{1d}, \quad v_2 = L_0 \dot{I}_{2d} + R I_{2d}. \tag{10}$$

This is possible if all the signals are determined off-line (open-loop). The allocation function from this class that results in the least energy consumption while keeping the voltages bounded by 10 volts is found to be the one with $F_0 = 1.095$. Under this allocation function, the energy consumption in a half period is $E = 4.3379c_e \times 10^{-3}$, slightly greater than the optimal value $4.3175c_e \times 10^{-3}$. In Fig. 6, the current signals and the voltage signals under this static allocation are plotted as compared to the optimal signals (in dashed curves). We see that the two sets of signals are very close.

This example shows that a static allocation can achieve almost the same minimal energy consumption as that by the optimal (non-static) allocation.

5 CONCLUSIONS

Power loss reduction and performance improvement are conflicting objectives. In this paper, we investigated the fundamental reason behind performance degradation when single actuator allocation strategy is adopted. Based on our investigation results, we formulated the problem of minimizing

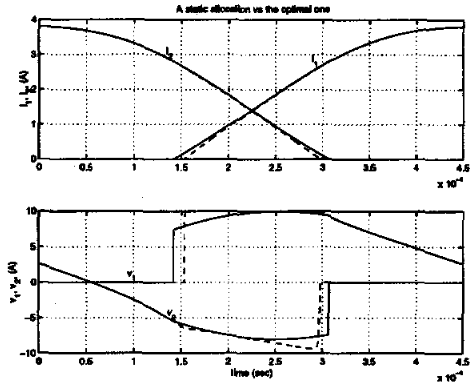

Figure 6: A static allocation as compared with the optimal one

power-loss under the constraint of bounded voltages. Optimal solution was obtained for the class of sinusoidal force signals. We also presented some static allocation strategies that would result in a suboptimal power-loss.

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