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Cancellation of Static and Sinusoidal Disturbance Forces in a Magnetical Suspension System Using Exerted Force and Flux Feedback

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ABSTRACT

This study is concerned with static and sinusoidal disturbance rejection for a single periodic input disturbance with known period and its experimental application to a magnetically suspended balance beam. In the area of active elimination of a disturbance force, the control input should have two different kinds of gains: one is to deliver a stable control and the other is a force component to cancel the external disturbance force. In this paper we employ a simple state feedback control law to make the balance beam stable and employ a linear observer to estimate the states which represent the external disturbance force components. In order to increase the tracking performance using the linear observer we present a flux feedback approach. Simulation results verify our proposed control method to reject a static and sinusoidal disturbance force.

INTRODUCTION

In magnetic bearing systems, one of the interesting research areas is to control of the electromagnet force to reject a static external disturbance force or a sinusoidal disturbance force which is produced by rotor mass imbalance or by a certain external disturbance force. Static disturbance force can be rejected by using a simple integrator even if the control law does not have a component for the cancellation of the static disturbance force. However if this static disturbance force is combined with a periodic force which has a certain frequency the simple integrator cannot reject the periodic disturbance term. In order to reject the periodic disturbance force, if the additional analog circuit which can reject the periodic force is not employed, the control law should have a certain component to cancel out the periodic disturbance force.

In this paper we present a method to reject a static and a periodic disturbance force which has a certain frequency. A magnetically suspended balance beam is used as a test rig. Static and sinusoidal disturbance forces are produced by a small fan which is placed on the one side of the balance beam. A state feedback control law is then employed to make the balance beam stable.

Since we do not employ any additional analog circuit, the state feedback control law should have a certain component to cancel out the disturbance term. The component included in the control law depends upon the disturbance term. Thus the effective rejection property of the disturbance force by using pure digital control method can be achieved by using a very accurate and high performance estimation or tracking tool for the disturbance force. In this paper we employ a linear observer to track the balance beam sensor output. Inherently the linear observer has some limited performance for a nonlinear system such as the magnetically suspended system. For example, unknown initial states of the magnetically suspended nonlinear system can degrade the estimation or tracking capability of the linear observer. This property of the linear observer, when it is employed to estimate the states of the magnetically suspended system, makes a control designer invest a lot of time into the observer feedback gain tuning.

In this paper we present a flux feedback approach to produce control current. The control current has two components. One is the gap deviation signal which is sensed by the gap sensor, the other is the deviation flux feedback signal which is expressed by a linear combination of the total force of the magnetic bearings. At the equilibrium point the total force of the magnetic bearings should be equal to the sum of the state feedback control effort and the estimated disturbance force. The deviation flux feedback signal can be expressed by the sum of the state feedback control effort and the estimated disturbance force. By using this method we can increase the performance of the linear observer to estimate the balance beam states which include external disturbance components due to the reduced nonlinearity which comes from flux feedback. First, we show the geometrical structure of the balance beam and mathematical model which includes static and sinusoidal disturbance components, and then we present how to design the linear observer. Finally we verify the proposed external disturbance cancellation method by simulation results.

BALANCE BEAM MATHEMATICAL MODEL

Fig. 1 shows the geometry of the symmetric balance beam with two horseshoe shaped magnetic bearings. Table 1 shows each parameter of the balance beam system. A small fan is placed on the one side of the balance beam to produce a static and periodic disturbance force.

TABLE I
BALANCE BEAM PARAMETERS

Parameter	Symbol	Value	Units
Angular Position	θ		rad
Half Bearing Span	L_a	0.1412	m
Mass Moment of Inertia about the Pivot Point	J	0.0948	kg m^2
Coil Current in Bearing 1	i_1'		A
Coil Current in Bearing 2	i_2'		A
Coil Resistance	R	0.7	Ω
Coil Inductance	L	0.728	mH
Magnetic Bearing Open Loop Stiffness	K_x	2114	N/m
Actuator Current Gain	K_i	1.074	N/A
Steady Current	i_0	1	A
Steady Gap	g_0	380	μm

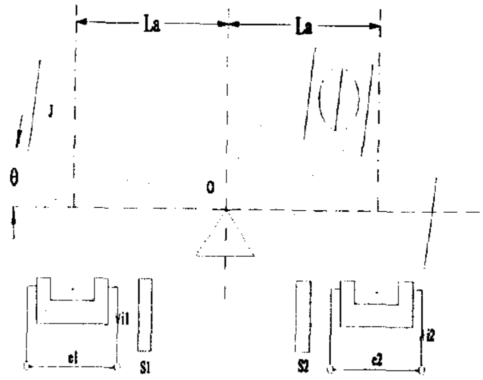

Fig. 1. Symmetric balance beam on two magnetic bearing and a small fan producing static and periodic disturbance

Equation of motion of the balance beam is expressed by the second order dynamic equation as:

$$J\ddot{\theta} + C_z\dot{\theta} = L_a(f_1 - f_2) + f_d \qquad (1)$$

where C_z is a damping factor of pivot and

$$f_1 = \mu_0 A_g N^2 \frac{(i_0+i_1')^2}{(2g_0+L_a\theta)^2}, \quad f_2 = \mu_0 A_g N^2 \frac{(i_0+i_2')^2}{(2g_0-L_a\theta)^2}$$

By linearization at $i_1' = 0$, $i_2' = 0$ and $\theta = 0$ equation (1) becomes

$$\dot{x} = Ax + Bu + Dw \qquad (2)$$

where

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ \frac{2k_z L_a^2}{J} & -\frac{C_z}{J} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{k_i L_a}{J} \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix}$$

As we see in Eq. (2) we employed a current control method for the rejection of the disturbance force acting on the balance beam. In (2) if we define w is a static and periodic disturbance terms w can be expressed such as:

$$w = k_1 + k_2 \sin(\beta t + \phi) + k_3 \cos(\beta t + \phi) \qquad (3)$$

where k_1, k_2 and k_3 are the amplitude of the disturbance force and β, ϕ represent a frequency and a phase of the periodic disturbance forces. Thus (3) becomes

$$w = C_w x_w \qquad (4)$$

where

$$x_w = \begin{bmatrix} 1 \\ \sin(\beta t + \phi) \\ \cos(\beta t + \phi) \end{bmatrix}, \quad C_w = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$$

Let states of the disturbance forces x_w be

$$\dot{x}_w = A_w x_w \qquad (5)$$

The solution of (5) is

$$x_w = C e^{A_w t} \qquad (6)$$

The differentiation of (6) yields

$$\dot{x}_w = A_w C e^{A_w t} = \begin{bmatrix} 0 \\ \beta\cos(\beta t + \phi) \\ \beta\sin(\beta t + \phi) \end{bmatrix} \qquad (7)$$

From (7) we get A_w matrix as:

$$A_w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \beta \\ 0 & -\beta & 0 \end{bmatrix}$$

The augmented state equation is derived by using (2), (4) and (5) with the result

$$\begin{bmatrix} \dot{x} \\ \dot{x}_w \end{bmatrix} = \begin{bmatrix} A & DC_w \\ 0 & A_w \end{bmatrix} \begin{bmatrix} x \\ x_w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \qquad (8)$$

$$y = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} x \\ x_w \end{bmatrix} \qquad (9)$$

DESIGN of DISTURBANCE ESTIMATOR

How to design a linear observer is already a well known procedure. In this section we simply show the design procedure of a linear observer for output tracking. From (8) and (9) we get the state space equations as:

$$\dot{x} = A_a x + B_a u \qquad (10)$$

$$y = C_a x \qquad (11)$$

The objective is to make $\lim_{t \to 0} \hat{y}\text{-}y = 0$ by using a certain observer feedback gain L. Here $\hat{y}$ represents estimated output. Thus the state space equation of the estimated system by observer (see Fig. 2) is

$$\begin{aligned} \dot{\hat{x}} &= A_a \hat{x} + B_a u - L(\hat{y} - y) \qquad (12) \\ &= A_a \hat{x} + B_a u - LC_a(\hat{x} - x) \qquad (13) \end{aligned}$$

where C_a=[1 0 0 0 0]. In this equation the estimator is expressed by the linear state space equations. However the balance beam supported by magnetic bearings has a highly nonlinear property, even if the mathematical model is expressed by the linear state space equations. The difference between the linear property of the estimator and nonlinear property of the system may cause difficulty in the design of a proper estimator or may not guarantee the performance of the estimator. In order to overcome this inherent drawback of the linear estimator we propose a novel control approach which will be introduced in the next section.

SYNTHESIS of FEEDBACK CONTROLLER

In the previous section we showed the normal design procedure of a linear observer. This linear observer has a limited estimation range due to the balance beam nonlinearity. The purpose of this section is to show a new approach using flux to estimate the plant states in the presence of plant nonlinearity. As we see in the fundamental equation flux produced by core magnet is proportional to the pole face area as:

$$\begin{aligned} \Phi &= BA_g \\ &= \frac{\mu_0 Ni}{2g}A_g = k_\phi \frac{i}{g} \end{aligned} \quad (14)$$

where $k_\phi = \frac{\mu_0 N A_g}{2}$. From (14) we get the current equation as a function of gap displacement and air gap flux.

$$i = \frac{1}{k_\phi} g\Phi \quad (15)$$

In the presence of external disturbance force the role of the electromagnet actuators is to reject the external disturbance force. The following formula meets the requirement:

$$F_n = F_1 - F_2 = -\hat{F}_d - kx \quad (16)$$

where $\hat{F}_d$ is the estimated disturbance force, k is the state feedback gain. Eq. (15) and (16) allow us to drive the control current as a function of the exerted force F_n and air gap flux Φ such as:

$$\begin{aligned} F_n &= k_f\left(\frac{i_1^2}{g_1^2} - \frac{i_2^2}{g_2^2}\right) \quad \left(k_f = \frac{\mu_0 N^2 A_g}{4}\right) \quad (17) \\ &= \frac{k_f}{k_\phi^2}(\Phi_1^2 - \Phi_2^2) \\ &= \frac{k_f}{k_\phi^2}[(\phi_b + \phi')^2 - (\phi_b - \phi')^2] \\ &= \frac{k_f}{k_\phi^2} 4\phi_b\phi' \end{aligned}$$

Eq. (17) yields

$$\phi' = \frac{F_n k_\phi^2}{4k_f\phi_b} \quad (18)$$

The air gap flux in each actuator is

$$\begin{aligned} \Phi_1 &= \phi_b + \phi' \\ &= \phi_b + \frac{F_n k_\phi^2}{4k_f\phi_b} \end{aligned} \quad (19)$$

$$\begin{aligned} \Phi_2 &= \phi_b - \phi' \\ &= \phi_b - \frac{F_n k_\phi^2}{4k_f\phi_b} \end{aligned} \quad (20)$$

Finally we get the control current from (15), (19) and (20) as:

$$\begin{aligned} i_1 &= \frac{1}{k_\phi} g_1 \Phi_1 \\ &= \frac{1}{k_\phi} g_1 \left(\phi_b + \frac{F_n k_\phi^2}{4k_f\phi_b}\right) \end{aligned} \quad (21)$$

$$\begin{aligned} i_2 &= \frac{1}{k_\phi} g_2 \Phi_2 \\ &= \frac{1}{k_\phi} g_2 \left(\phi_b - \frac{F_n k_\phi^2}{4k_f\phi_b}\right) \end{aligned} \quad (22)$$

In (21) and (22) the exerted force is implemented by the following relation.

$$F_n = -\hat{F}_d - kx \quad (23)$$

In the above procedure, we showed a novel synthesis approach of the control current in the presence of the external static and sinusoidal disturbance force. The main frame in this procedure is in the equation (14) which represents a linear combination, Φ is proportional to $\frac{i}{g}$. Based upon this linear combination the control current is also expressed by the linear multiplication of Φ and g, and then the final control current is achieved by the modified formula which has the deviated flux feedback components as shown in Eq. (21) and (22). As we have mentioned, Eq. (21) and (22) have the exerted force components that involve estimated disturbance force by the observer and state feedback. Thus, once the observer estimates the external disturbance force $\hat{F}_d$, by substituting (21) and (22) into (2) we can easily check the cancellation of the external disturbance force term. The performance of the estimator is also increased by the feedback of exerted force component which is included in the deviated flux due to the reduced nonlinear property in the control procedure. Fig. 2 shows the block diagram for the rejection of the external disturbance force using observer.

SIMULATION RESULTS

In this section we show the simulation results of the static and periodic disturbance force rejection. A MATLAB Simulink model was designed based upon the nonlinear electromagnet force equation, which

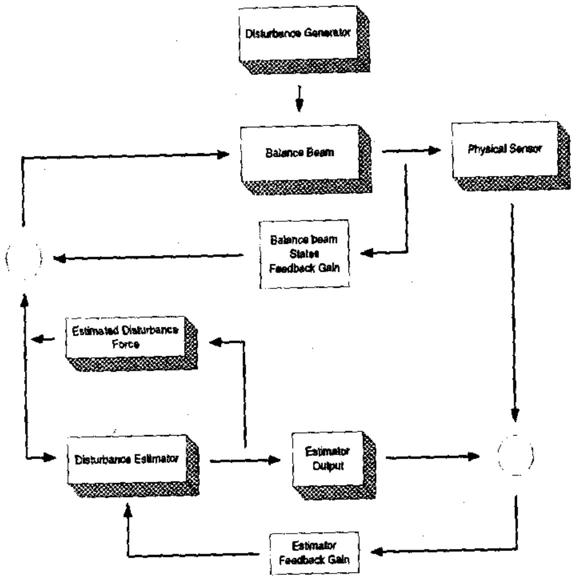

Fig. 2. Block diagram for disturbance estimation

represents a nonlinear simulation. For the simulations we used the following feedback gains and parameters of the static and sinusoidal disturbance.

• State feedback gains :
K_p=5500, K_d=500

• Estimator feedback gains :
$10^6 \times [0.0008\ \ 0.2006\ \ 0.0983\ \ 2.7952\ \ -0.3089]^T$

• Disturbance coefficient matrix :
C_w=[5 1 1]

• Frequency of the sinusoidal disturbance :
β=60[rad/sec]

• Phase of the sinusoidal disturbance :
ϕ=10[rad]

Fig. 3 and 4 show the balance beam gap deviations and the static and sinusoidal disturbance force which has 10[Hz] frequency and 10[rad] phase difference produced by small fan. In order to realize the proposed control method we need to know the exact value of the phase difference in the small fan. Only for the simulation we set 10[rad] for the phase of the periodic function. From Fig. 3 and 4 we see good rejection of the external static and periodic disturbance force. The peak in Fig 3. occurs due to the static disturbance as we see in Fig. 4. and the disturbance force estimation error in the transient state of the estimator. Fig. 5 represents the exerted input force to produce the control current expressed by Eq. (21) and (22). If we compare Fig. 5 with Fig. 4 we see the same amplitude in the static and sinusoidal disturbance force except for the sign convention. This means that the exerted force components included in the control current rejects the disturbance force. Fig. 6 shows the simulation results for the disturbance force estimated by the designed observer. In this figure we see good tracking properties of the estimator after the very short transient state. The transient state has a certain unexpected

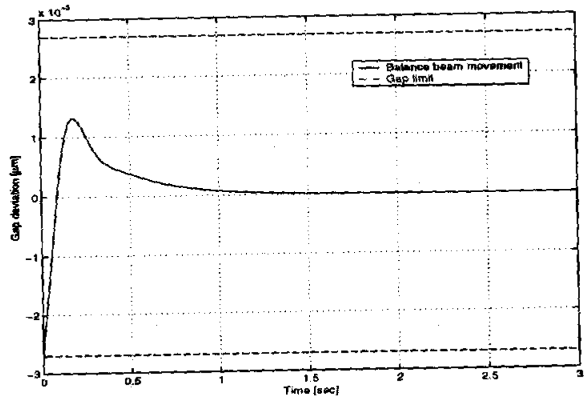

Fig. 3. Gap deviations

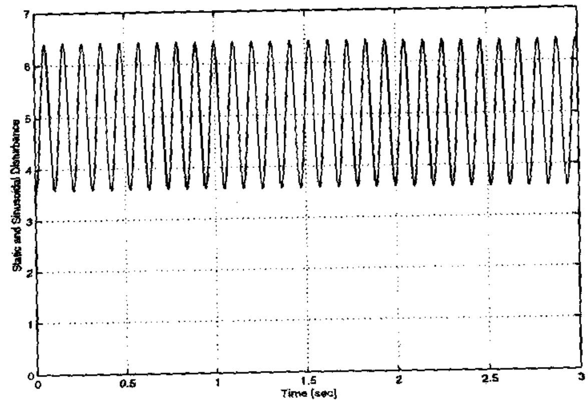

Fig. 4. Static and sinusoidal disturbance

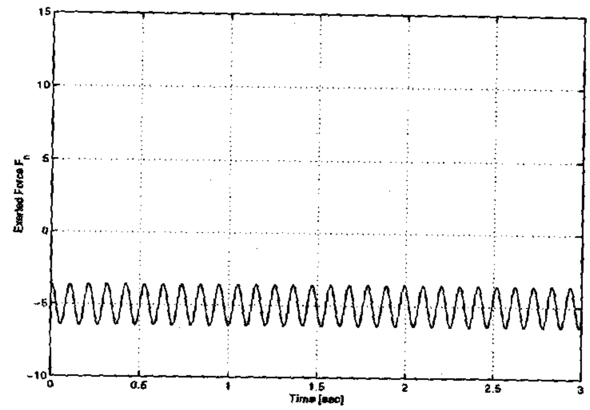

Fig. 5. Exerted input force for current

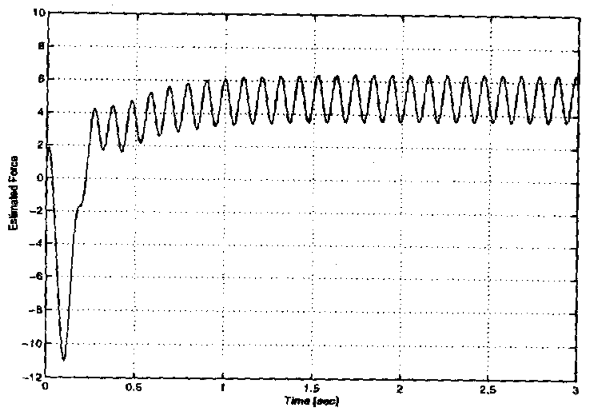

Fig. 6. Estimated disturbance force

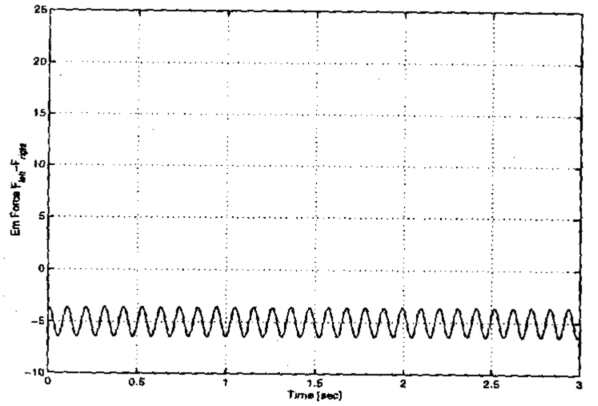

Fig. 7. Electromagnet actuator force: $F_1 - F_2$

response due to the balance beam nonlinearity and the linear property of the estimator. From the implementation point of view the estimator feedback gains should be selected very carefully to avoid the unexpected transient response above mentioned. Fig. 7 shows the force produced by the electromagnets actuators. From these figures we see the effort to reject the external disturbance force in the actuators.

CONCLUSIONS

In this paper we proposed a control method using an exerted force and flux feedback to reject a static and periodic disturbance force. A new control current formula including exerted force component and deviated flux component was achieved by a combining the state feedback control law and the output of the estimator. First we showed the balance beam geometrical scheme and the fundamental equation of motion of the balance beam, and then we showed the synthesis procedure of the control current to cancel out the external disturbance force. Finally the proposed control approach was validated by the simulation results produced by a nonlinear simulation model.

REFERENCES

1. Benjamin C. Kuo, "Automatic Control System", *PRENTICE-HALL.*

2. Kenzo Nonami, Qi-fu Fan, Hirochika Ueyama, "Unbalance Vibration Control of Magnetic Bearing Systems Using Adaptive Algorithm with Disturbance Frequency Estimation", *6th ISMB, August 5-7, 1998.*

3. Tingshu Hu, Zongli Lin, "Control Systems with Actuator Saturation: Analysis and Design", *Birkhauser.*

4. P. K. Sinha, "Electromagnetic Suspension: Dynamics and Control', *IEE CONTROL ENGINEERING SERIES 30.*

5. "Magnetic Suspension Technology: Magnetic Levitation System and Magnetic Bearings", *CORONA PUBLISHING CO.*

6. Jun-Ho Lee, P.E. Allaire, Gang Tao X. Zhang , "Integral Sliding Mode Control of a Magnetically Suspended Balance Beam: Analysis , Simulation and Experimentation", *IEEE Trans. on Mechatronics, Vol. 6, No. 3, pp. 338-346, 2001.*