8th International Symposium on Magnetic Bearing
August 26-28, 2002, Mito, Japan

A Convex Optimization Approach to Robust Controller Design for Active Magnetic Bearing Suspension Systems

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ABSTRACT

Active magnetic bearings (AMBs) which suspend high speed rotor systems present a challenge in control design. A controller with robustness to uncertainty and capable of adjusting itself according to the rotor speed is essential. Based on a prototype for a flywheel energy storage system constructed at University of Virginia, a convex approach to design a robust linear parameter varying (LPV) controller is proposed. As a result, the robust LPV controller can maintain its stabilizing capability and robust performance for the whole speed range. One impediment for implementing the LPV controller is the computational costliness. Based on the convex approach, an effective methodology for implementing the LPV controller on-line is proposed. Simulation and experimental results are in agreement with the theoretical derivation.

Keywords: Magnetic bearings, robust, LPV, convex optimization, implementation

1 INTRODUCTION

Magnetic bearings suspended rotating systems have long been considered a promising technology due to their inherent advantages over traditional ball bearing systems. Typically, such a system contains a high-speed spinning rotor which is supported by active magnetic bearings (AMBs). The control of the system has never been trivial. Because of the gyroscopic effects in the spinning rotor, the system dynamics can change significantly as rotor speed varies. The gyroscopic effects are linear in the rotational speed, which results in an LPV system model. Different control strategies have been proposed to address this problem. One is to consider the varying rotational speed as an uncertainty of the model, as in μ synthesis. This method is effective in constructing good controllers for a specific speed range, but the performance will deteriorate outside this range [1]. Another approach is the traditional gain-scheduling LPV controller which depends upon a gridding method to deal with uncertainty [2, 3, 5]. The gridding method may lead to numerical problems and implementation difficulties. In [9], a method of formulating unmeasured structured uncertainty into LPV framework has been proposed. In this paper, we continue on the work of [9] and address various issues in controller implementation. The system is characterized as a convex set specifying both structured uncertainty and the linearly varying parameter (rotor rotational speed p). The control objective is formulated as a convex optimization problem. The final LPV controller is robustly stable at each speed and meets the H_∞ norm specification at any rotational speed. To make the high order LPV controller feasible in real-time implementation, several issues have been addressed. An off-line LPV discretization method is proposed. The expensive on-line discretization is saved and the resulting discrete controller performance is comparable with the continuous time LPV controller.

The remaining part of this paper is organized as follows. In Section 2, we introduce the model of the flexible rotor supported on AMBs, which is based on the test rig constructed at the University of Virginia (UVa). In Section 3, we present the analysis and design methodology for robust LPV control. In Section 4, we discuss several issues on the LPV controller implementation and give the off-line LPV discretization method. A brief concluding remark is made in Section 5.

2 PLANT MODEL

The AMB control test rig at UVa is constructed to simulate a flywheel energy storage system (Fig. 1). The rotor is located vertically, about 40 lb in weight and 34 in in length. A thrust magnetic bearing is used to levitate the rotor in the axial direction. In two radial directions, we use two pairs of active magnetic bearings to constrain the position of the spinning rotor. Two pairs of sensors measure the displacements of the rotor from its geometric center. Thus, it is a four input (bearing currents), four output (displacements) multi-input

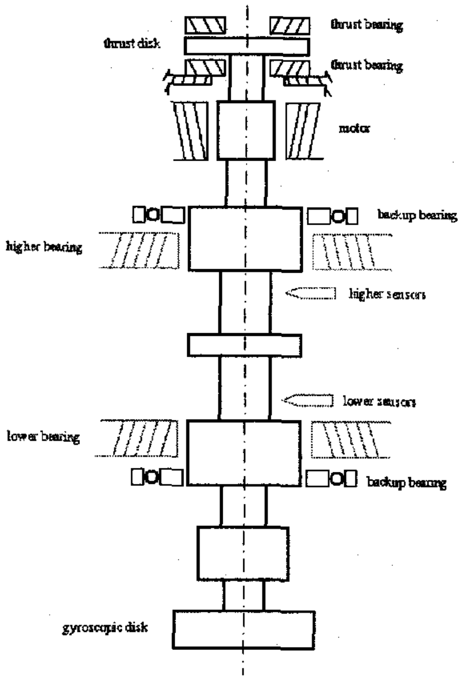

Figure 1: Schematic of the rotor

multi-output (MIMO) system. In addition, the high speed rotor displays significant gyroscopic effects and the dynamic model is in an LPV form, i.e., the system matrix is $A(p) = A_0 + pA_p$, where p is the rotor speed. Thus, to take advantage of the LPV controller design, the rotor speed is also measured on-line for feedback into the controller. By means

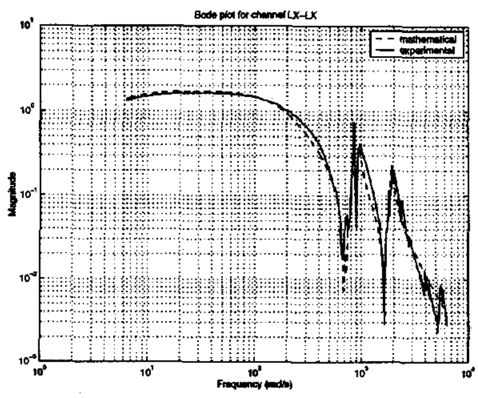

Figure 2: Bode plot for one channel (mathematical model v.s. experimental result)

of finite element modeling (FEM) and modal test for rotor dynamics, we construct a nominal system model with 28 states, 4 inputs and 4 outputs, in an LPV form [6]. The mathematical model frequency response matches the experimental system response very well up to the third flexible mode of the rotor in the operating frequency range. (Fig. 2). The nominal model provides a good estimate of the complex behavior of system dynamics, but it is not perfect. For our robust control design, we have to take into account the uncertainty of the plant. The following structured uncertainties are critical in our system design:

- uncertainty of natural frequencies
- magnetic bearing parameter uncertainty
- sensor and amplifier gain uncertainty

Based on the nominal plant, the uncertainties described above can be formulated in a linear fraction transformation (LFT) form, which results in the augmented plant with a 34×34 diagonalized structured uncertainty block.

For robust control design, effective weighting functions and performance specifications are also indispensable. For our specific problem, we specify the following objectives:

- disturbance rejection
- noise attenuation
- regulation error minimization
- control effort restraint
- unstructured uncertainty characterization

Our weighting functions are constructed and tuned by these criteria. This problem can be formulated as a weighted H_∞ control problem.

3 CONTROLLER SYNTHESIS

Our robust LPV controller design will be described in the following steps. We first formulate the problem as a combination of robust and gain-scheduled control, then propose a convex approach for controller synthesis, and finally give the simulation results.

3.1 Problem formulation

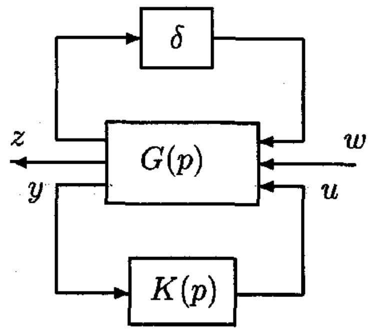

Figure 3: Robust LPV control

The augmented system model can be written as:

$$\left\{ \begin{array}{rcl} \dot{x} & = & (A_\delta(\delta) + pA_p)x + B_1 w + B_2 u, \\ z & = & C_1 x + D_{11} w + D_{12} u, \\ y & = & C_2 x + D_{21} w, \end{array} \right. \quad (1)$$

where $A_\delta(\delta) = A_0 + \delta_1 A_1 + \cdots + \delta_r A_r$ can be considered as affine function of the parameter vector $\delta = (\delta_1, \delta_2, \cdots, \delta_r) \in \Delta$ with fixed coefficient matrices $A_0, A_1, \cdots, A_r$, where $\Delta := \{(\delta_1, \delta_2, \cdots, \delta_r) : \delta_i \in \{\underline{\delta}_i, \bar{\delta}_i\}\}$ is the set of structured uncertainty range. For the robust stability problem, we are seeking a controller that can stabilize the closed loop system for any fixed vector δ in the predefined uncertainty range Δ. Moreover, the gyroscopic part is linearly dependent on $p \in (p_1, p_2) = (0, p_{\max})$, which is measured in real time and can be fed to the controller to optimize the performance of the closed loop system over the whole range of the rotor speed. All these properties entail an LPV gain-scheduled robust controller design. Our design objective is formulated as follows. For any given $\gamma > 0$, construct a feedback law of the form

$$\left\{ \begin{array}{rcl} \dot{x}_c &=& A_c(p)x_c + B_c(p)y, \\ u &=& C_c(p)x_c + D_c(p)y, \end{array} \right. \tag{2}$$

such that, for each $\delta \in \Delta$ and $p \in (0, p_{\max})$, the closed-loop system is asymptotically stable and the H_∞ norm of its transfer function from the external disturbance w to the controlled output z is less than or equal to γ. This problem is a combination of robust and gain-scheduled control, but can be formulated as convex optimization problem using LMI techniques.

3.2 Convex characterization

The set of all possible values of $\delta \in \Delta$ and $p \in (0, p_{\max})$ can also be described by a convex hull

$$\mathrm{co}\{v_1, v_2, \cdots, v_n\}$$

$$= \left\{ v = \begin{pmatrix} \delta \\ p \end{pmatrix} \in \Re^{r+1} : v = \sum_{i=1}^{n} \alpha_i v_i, \alpha_i \in (0,1), \sum_{i=1}^{n} \alpha_i = 1 \right\},$$

where $n = 2^{r+1}$ is the number of vertices. Now consider the mapping from the parameter δ and p to the system matrices. The system matrices also specify a convex hull, i.e.,

$$\begin{pmatrix} A(\delta,p) & B \\ C & D \end{pmatrix} := \begin{pmatrix} A_\delta(\delta) + pA_p & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{pmatrix} \in \Omega$$

$$:= \mathrm{co}\left\{ \begin{pmatrix} A_i & B \\ C & D \end{pmatrix} := \begin{pmatrix} A(v_i) & B \\ C & D \end{pmatrix}, i = 1, \cdots, n \right\}.$$

Let

$$A_\Omega := \mathrm{co}\{A(v_i), i = 1, 2, \cdots, n\}.$$

Each element of A_Ω can be represented by

$$\alpha_{p_1} \sum_{i=1}^{2^r} \alpha_{\delta_i} A_{p_1 i} + \alpha_{p_2} \sum_{i=1}^{2^r} \alpha_{\delta_i} A_{p_2 i},$$

where

$$\alpha_{p_1} = \frac{p_2 - p}{p_2 - p_1} \in (0,1), \quad \alpha_{p_2} = \frac{p - p_1}{p_2 - p_1} \in (0,1)$$

satisfy $\alpha_{p_1} + \alpha_{p_2} = 1$, $\alpha_{\delta_i} \in (0,1)$ satisfy $\sum_{i=1}^{2^r} \alpha_{\delta_i} = 1$, and $A_{p_1 i}$, $A_{p_2 i}$ are constant matrices that depend on $\underline{\delta}_i$, $\bar{\delta}_i$ and p_1, p_2. By virtue of convexity, the whole LPV uncertainty plant H_∞ control is tractable by applying H_∞ control rules to these vertices based on a convex approach. For a given $\gamma > 0$, we denote the H_∞ controller at a single vertex $\begin{pmatrix} A(v_i) & B \\ C & D \end{pmatrix}$ as $\mathcal{K}_i = \begin{pmatrix} A_{ci} & B_{ci} \\ C_{ci} & D_{ci} \end{pmatrix}$. The transfer function of closed-loop system at the vertex can be represented as

$$\begin{aligned} T_{zwi} = G_i(s) &:= \begin{pmatrix} A_{vi} & B_{vi} \\ C_{vi} & D_{vi} \end{pmatrix} \\ &= \begin{pmatrix} \mathcal{A}_i & \mathcal{B}_1 \\ \mathcal{C}_1 & \mathcal{D}_{11} \end{pmatrix} + \begin{pmatrix} \mathcal{B}_2 \\ \mathcal{D}_{12} \end{pmatrix} \mathcal{K}_i \left(\mathcal{C}_2 \ \mathcal{D}_{21} \right), \end{aligned} \tag{3}$$

where,

$$\begin{pmatrix} \mathcal{A}_i & \mathcal{B}_1 & \mathcal{B}_2 \\ \mathcal{C}_1 & \mathcal{D}_{11} & \mathcal{D}_{12} \\ \mathcal{C}_2 & \mathcal{D}_{21} & 0 \end{pmatrix} := \left(\begin{array}{cc|c|cc} A_i & 0 & B_1 & 0 & B_2 \\ 0 & 0 & 0 & I & 0 \\ \hline C_1 & 0 & D_{11} & 0 & D_{12} \\ \hline 0 & I & 0 & 0 & 0 \\ C_2 & 0 & D_{21} & 0 & 0 \end{array} \right).$$

The closed-loop system has the following properties:

- A_{vi} is Hurwitz,
- $\|G_i\|_\infty < \gamma$.

Theorem 1 *Consider the closed-loop system at a single vertex as in (3). Let $\gamma > 0$ be given. Then the following statements are equivalent.*

1. *A_{vi} is Hurwitz and $\|G_i\|_\infty < \gamma$;*
2. *There exists a real matrix $P = P^{\mathrm{T}} > 0$ such that*

$$\begin{pmatrix} PA_{vi} + A_{vi}{}^{\mathrm{T}}P & PB_{vi} & C_{vi}{}^{\mathrm{T}} \\ B_{vi}{}^{\mathrm{T}}P & -\gamma I & D_{vi}{}^{\mathrm{T}} \\ C_{vi} & D_{vi} & -\gamma I \end{pmatrix} < 0. \tag{4}$$

This theorem provides a convex approach to analyzing and designing an H_∞ controller. For our system, the parameter δ in the plant is unmeasured, but p is available on line. Our objective is to design an LPV gain-scheduled controller which is asymptotically stabilizing for each $\delta \in \Delta$ at every point of the trajectory of p. For the convex hull $\Omega = \mathrm{co}\{A_{p_1 i}, A_{p_2 i}, i = 1, 2, \cdots, 2^r\}$ with vertices $[A_{p_1 i}, A_{p_2 i}, i = 1, 2, \cdots, 2^r]$, we denote the corresponding $\mathcal{A}$ as $[\mathcal{A}_{p_1 i}, \mathcal{A}_{p_2 i}, i = 1, 2, \cdots, 2^r]$. Denote the corresponding common H_∞ controller for vertices $A_{p_1 i}$ and $A_{p_2 i}$ as $\mathcal{K}_{p_1}$ and $\mathcal{K}_{p_2}$ respectively. Then the desired LPV robust controller is in the form

$$\mathcal{K}(p) := \alpha_{p_1} \mathcal{K}_{p_1} + \alpha_{p_2} \mathcal{K}_{p_2}. \tag{5}$$

Theorem 2 [9] *Consider the convex hull of the plant Ω and the feedback control law $\mathcal{K}(p)$. the following statements are equivalent:*

- *For any $\delta \in \Delta$ and $p \in (0, p_{\max})$, the closed-loop system is asymptotically stable and $\|G\| < \gamma$;*
- *There exists a single real matrix $P = P^{\mathrm{T}} > 0$ satisfying (4) for $i = 1, 2, \cdots, n$.*

3.3 Controller synthesis

Using the Projection Lemma [4] and taking advantage of the convexity characterization, we construct a real matrix $P > 0$ in Theorem 2 for all the vertices of the convex hull, then solve (4) for $\mathcal{K}_{p_1}$ and $\mathcal{K}_{p_2}$ at the vertices of Ω. The final robust gain-scheduled LPV controller is (5).

3.4 Simulation

To confirm the robustness of the controller, we use μ analysis to test the robust stability in face of the structured uncertainty over different speed points. For a 3% uncertainty ratio, the nominal performance is well bellow 1 (our desired γ is 1). The μ bound is below 1 for most of the speed range (our designed speed range is $p \in (0, 2000)$ rad/s, (see Fig. 4). When we implement the controller on the test rig, it is successfully levitated. To test the closed-loop performance, we perform Sine-Sweep Test, which imposes an external sinusoidal disturbance signal into the system input, the frequency response is comparable with the simulation bode plot (Fig. 5).

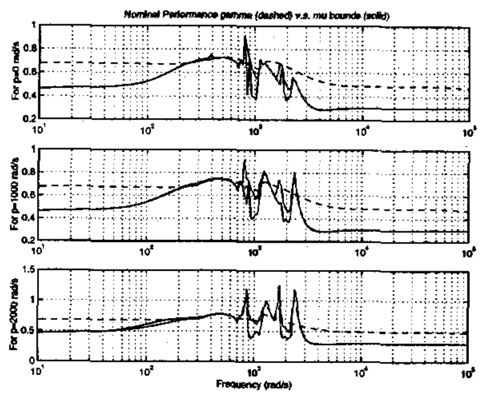

Figure 4: Nominal performance comparison

4 CONTROLLER IMPLEMENTATION

We have arrived at an LPV controller, i.e.,

$$\begin{cases} \dot{\xi} &= A(p)\xi + By, \\ u &= C\xi + Dy. \end{cases} \qquad (6)$$

One impediment for implementing such an LPV controller is the computational costliness. Assume that

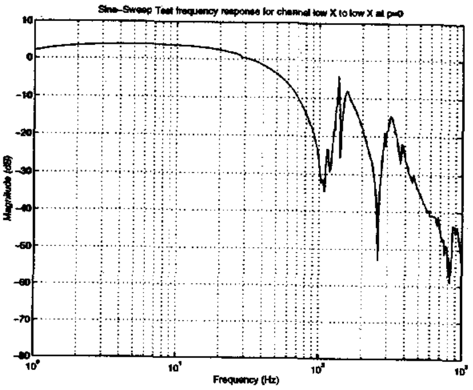

Figure 5: Sine sweep test for one input output channel

the controller is in state $\xi(kT) = \xi_k$ at time kT, while

$$p(t) = p_k; \quad y(t) = y_k \quad \text{for} \quad kT \leq t < (k+1)T,$$

then for $t \geq kT$,

$$\xi(t) = e^{A(p_k)(t-kT)}\xi_k + \left[\int_{kT}^{t} e^{A(p_k)(t-\tau)} B d\tau\right] y_k.$$

In particular, for ξ_{k+1} at $t = (k+1)T$

$$\begin{cases} \xi_{k+1} = e^{A(p_k)T}\xi_k + \left[\int_0^T e^{A(p_k)\tau} B d\tau\right] y_k, \\ u_k = C\xi_k + Dy_k. \end{cases} \qquad (7)$$

Because of the matrix exponential and the matrix integral, the computation load will increase exponentially with the increase of the controller order and the sampling rate. Thus, the real time implementation of such a discrete LPV controller is a challenge, especially for high order high sampling rate applications. To fully exploit the rapid developing processor and memory technology, there are lots of issues that need to be considered in both software and hardware respects. These include:

- sampling rate
- analogue-to-digital (A/D) conversion
- computer arithmetic (floating or fixed point)
- word-length
- memory requirements
- computational delay

We will briefly discuss some issues in our controller implementation.

4.1 Sampling rate selection and anti-aliasing

In our design, we first design a continuous controller based on the continuous time plant model, then derive the equivalent discretized digital controller for implementation. Thus, the higher the sampling rate, the better the controller performs. However, the computation cost also increases because less time is available to process the controller calculations.

Based on the sampling theorem, we would like to have a sampling rate of over 600 Hz, which is greater than twice the frequency of the second flexible mode ($f_b \cong 300\ Hz$).

Another issue is the effect of the sampling rate on quantization error, which is due to the finite word length of the digital computer and A/D converter. As the sampling frequency increases, the roundoff noise will be more significant for the same word length [7]. In our case, we use floating point calculation in our digital computer, so the finite word length effects on the controller performance is not significant.

As a general rule of thumb, the sampling rate should be more than 5 times f_b, which is $1500\,Hz$. In our system, to get better approximation of the synthesized continuous controller, and consider our off-line discretization LPV method (Remark 4.2), we use a sampling rate of $8K\,Hz$.

To avoid aliasing, a low-pass filter, or anti-aliasing filter is used to reduce the high frequency components in sensor signals before the A/D conversion. From the control aspect, there will be a phase distortion or a time delay on the filter, which will have a negative effect on the system performance. We have two approaches to solving the problem. One is to use a higher sampling rate, so that the anti-aliasing filter will have a higher cutoff frequency and the phase distortion which is strongest around the cutoff frequency will not affect the sensor frequencies and system performance. The other is to model the phase delay and take it into account when designing the controller. In our case, we use a second order Pade approximation of time delays in system modeling.

For our LPV controller, an external signal (rotor speed p) is sampled on-line to for feedback into the controller. Ideally, we will sample it at the same rate as the feedback signal (rotor displacements), but practically we can lower this sampling rate and maintain the stability and performance.

4.2 Time constraint on computation

Because the time varying natural of the LPV controller, the controller dynamics need to be refreshed at each sampling instant. This might be a problem in real time implementation. Because the computation for discretization and matrix/vector calculation is generally too heavy to be completed in single sampling period. However, there are two approaches to alleviating the burden. One is to refresh the LPV controller at a lower rate than the sampling rate. The following theorem follows from a simple continuity argument.

Theorem 3 *For the plant (1) and the LPV controller (5) at any given point $p = p_0$, denoted as $\mathcal{K}(p_0)$, there exists scalar α such that the closed-loop system with LTI controller $\mathcal{K}(p_0)$ ensures both stability and the H-infinity norm bound γ for any p such that $p \in (0, p_{max})$ and $\|p - p_0\| < \alpha$.*

In practice, the difficulty lies in estimating the value of α. In our work, we resort to simulation. Once we have an estimate of the value of α, we can schedule the trajectory of the varying parameter p, divide the whole trajectory into smaller zones, and switch controller between adjacent zones. Thus, we can discretize the LTI controller for each zone off-line and save the time for online discretization. However, this is at the cost of memory requirements.

Another method of reducing the computation load is to do a similarity transformation on the discretized controller. For our discretized LTI controller $K(z)$,

$$\begin{aligned} K(z) &= C_c(zI - A_c)^{-1}B_c + D_c \\ &= C_cT(zI - T^{-1}A_cT)^{-1}T^{-1}B_c + D_c \end{aligned}$$

where T is a non-singular similarity transformation matrix. Thus, the realization of $K(z)$ is not unique. For a high order controller, on-line matrix/vector computations can be greatly reduced by increasing the sparseness of the dynamics matrix. One easy way is to diagonalize or almost diagonalize A_c into the Jordan canonical form. Then, there will be at least $n^2 - 3n + 2$ zeros in A_c of $n \times n$ dimension, so the computation and memory allocation can be reduced by around $\frac{n-3}{n}$ for large n. However, for a finite word length digital controller, some roundoff noise is inevitable, this transformation may not be good at minimizing roundoff noise and coefficient sensitivity [7, 8]. For our system, we use floating point representation, this problem is not so significant.

For the LPV controller, to update the controller dynamics, the computation load for on-line discretization at each refreshing point is almost unrealistic for high order controller, because it involves matrix exponential or matrix inversion. However, under some assumption, we can reduce the computation to only matrix/vector multiplications and additions, we call it off-line LPV discretization method. The idea is illustrated hereafter.

For our continuous LPV controller (6), one method for finding discrete-time equivalent is to integrate $\xi(t)$ as follows:

$$\xi(t) = \xi(t_0) + \int_{t_0}^{t} [A(p)\xi(t) + By(t)]\,dt,$$

For evenly spaced samples, at $t = kT, k = 0, 1, 2, \cdots$

$$\xi(kT + T) = \xi(kT) + \int_{kT}^{kT+T} [A(p)\xi(t) + By(t)]\,dt.$$

Assume T is very small, set

$\xi(kT) = \xi_k, p(t) = p_k, y(t) = y_k$ for $kT \le t < (k+1)T$, using Euler's rectangular approximation of the integral, the equivalent discrete time controller is of the form:

$$\begin{cases} \xi_{k+1} &= [I + A(p_k)T]\,\xi_k + BTy_k, \\ u_k &= C\xi_k + Dy_k. \end{cases} \quad (8)$$

From our controller synthesis,

$$A(p) = \alpha_{p1}A_{p1} + \alpha_{p2}A_{p2},$$

where A_{p1} and A_{p2} denote the dynamic matrices of $\mathcal{K}_{p_1}$ and $\mathcal{K}_{p_2}$ respectively. Then by convexity of our LPV controller, our discrete time equivalent can be written as

$$\mathcal{D}(p) := \alpha_{p_1}\mathcal{D}_{p_1} + \alpha_{p_2}\mathcal{D}_{p_2}, \quad (9)$$

where $\mathcal{D}_{p_1}$ and $\mathcal{D}_{p_2}$ are discretized equivalent for $\mathcal{K}_{p_1}$ and $\mathcal{K}_{p_2}$ respectively.

Remark 4.1 *When we use truncated Taylor expansion to approximate the matrix exponential in (7), we will get (8). So the accuracy of our discrete LPV controller synthesis also depends on the norm of $A(p)$. The lower the magnitude of $\|A(p)\|$, the better the approximation.*

Remark 4.2 *With our assumption on small T, obviously, the higher the sampling rate, the better the approximation. But the computation time for each sampling point will be less. It is a trade off.*

With the off-line discretization method, our computation load is comparable with an LTI controller. At the same time, the performance is also comparable with a regular on-line discretization LPV. Moreover, the assumption for our method is not a very strict constraint. Fig. 6 is the performance comparison between on-line discretization and our off-line discretization method in implementing the LPV controller. We can see that there is no obvious difference, but the computation load for our method is greatly reduced.

5 CONCLUSIONS

In this paper, we presented a convex approach to design robust LPV controller for active magnetic bearing suspension systems and discussed some issues on implementing the LPV controller. The robustness and performance of the controller are demonstrated and several methods to reduce the online implementation cost are given.

ACKNOWLEDGMENTS

The authors wish to acknowledge that this research was supported by AFS Trinity Power Corporation as an independently funded continuation of research originally undertaken as a NASA, Goddard Space Flight Center, Small Business Innovative Research (SIBR) project.

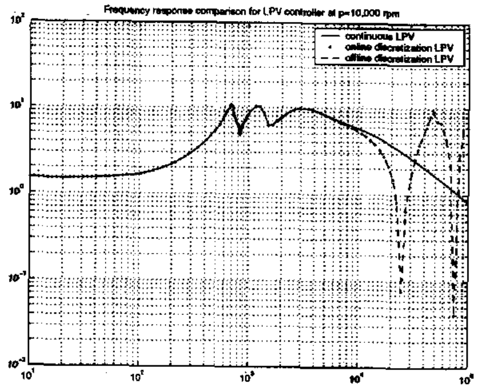

Figure 6: Performance of off-line LPV discretization method

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