

# ZERO POWER NONLINEAR CONTROL OF MAGNETIC BEARING SYSTEM

Kenzo NONAMI\*, Zi-he LIU\*\*

\*Department of Electronics and Mechanical Engineering, Faculty of Eng.,  
Chiba University E-mail: [nonami@meneth.tn.chiba-u.ac.jp](mailto:nonami@meneth.tn.chiba-u.ac.jp)

\*\*Artificial System Science Course, Graduate School of Science and Technology,  
Chiba University, E-mail: [liu@mec2.tn.chiba-u.ac.jp](mailto:liu@mec2.tn.chiba-u.ac.jp)

## ABSTRACT

Generally, the control strategies for a magnetic bearing often focus on the linear control techniques by through the bias current to a pair of the electromagnets and design a control system based on the linear control theory. In this paper, we propose a new algorithm to realize a nonlinear zero power control of rigid rotor-magnetic bearing system using only electromagnets by backstepping method which belongs to a nonlinear control theory. And we designed the control system using backstepping method for the reduced order model. In particular, we also propose a new algorithm which will be able to find out their control currents with a global stability guarantee. Furthermore, those methods were applied to the rigid rotor and the flexible rotor-magnetic bearing system to examine the stability by simulations and experiments, the results show the validity of the proposed method.

**Keywords:** Magnetic Bearing System, Backstepping Method, Zero-Power Control, Nonlinear Control, Rigid Rotor, Flexible Rotor, Reduced Order Model, Global Stability, Switching Control, Lyapunov Function, Experiment.

## INTRODUCTION

A magnetic bearing system consists of a couple of electromagnets which face each other, and it supports the rotor by magnetic force without any contact at all. It is possible for the rotor supported by a magnetic bearing to rotate with ultra-high-speed. So, it has a lot of advantages such as no wear and friction, no machinery energy loss, a little noise, and it needs no lubricating oil, so it is very clean for around environment etc. However, a magnetic bearing system is unstable essentially, the feedback control is indispensable. Since a magnetic force has a very strong

non-linearity, as for the conventional control technique, it is common to linearize a nonlinear system and design a control system based on linear control theory by supplying bias currents on a pair of electromagnets which face each other. However, the method must always continue supplying bias currents, even when the position of the rotor is at equilibrium point, and it has the fault that electric power consumption becomes large. Moreover, when the rotor was rotating, the eddy current loss occurs due to this bias currents. In order to reduce such loss, we usually apply a special devise using the lamination steel plate, but it makes a magnetic bearing more expensive actually. To utilization of the magnetic bearing to a field especially like the energy storage flywheel system, the zero power or the low power consumption type is inevitable from the viewpoint of energy income and outgo. When the rotor is rotating, the energy loss reduction also will become important subject.

Recently, the research of the zero-power nonlinear control which aims at the electric power reduction has centering on a magnetic bearing is beginning to be done. The zero power magnetic bearing which used the permanent magnet and the electromagnet together has been reported<sup>(1)</sup>. This magnetic bearing can reduce power consumption drastically. However, there is a problem to which the structure of an actuator become complicated with magnetic bearing only using the electromagnet, and the cost becomes more expensive. In the case that is only using electromagnet and not supplying bias currents, the system becoming nonlinear system, the application of the linear control theory will be restricted. The nonlinear control technique have been proposed<sup>(2)-(4)</sup>, but those method which used bias currents and not for realizing zero-power control.

The effective proposal to zero power control which

only use an electromagnet has been reported for the magnetic bearing<sup>(5)-(7)</sup>. In the case that is not used bias current, the switching type of zero power nonlinear control technique has been proposed based on Lyapunov direct method for the rigid rotor<sup>(5)(6)</sup>. Furthermore, as a result that it aims at the improvement of the control performance, the control system which satisfies the asymptotic stability condition of the Lyapunov method has been obtained numerically. The zero power nonlinear control technique based on backstepping method which belongs to the nonlinear control theory<sup>(7)</sup> has been proposed. Those validity are verified by experiments. Especially, the control system design method without supplying the bias current and guaranteeing the global stability has been proposed<sup>(7)</sup>. The switching type which makes an electromagnet off in the case that the rotor approached it, and the improvement type without switching has been proposed to avoid the problem that an input becomes an imaginary number. Furthermore, the servo system also have be designed to control state disturbance based on the backstepping method.

In the previous study, the nonlinear control of the subsystem without a couple was designed on the four degree of freedom model which considered that it is rigid mode and as simple as single degree-of-freedom which considered that the rotor is a independent. However, we must deal the some magnetic bearing system such as flexible rotor actually. In this study, we proposed a new zero power nonlinear control algorithm which is different from reference<sup>(7)</sup> in order to control the rigid mode, and also designed the control system for the reduced order model based on the backstepping method. Furthermore, we applied the method to examine the stability of the flexible rotor-magnetic bearing system, and the validity was shown by simulation. Especially, we proposed a new method to find out the control input current of each electromagnet after that the virtual input of the linear system was found out.

## CONTROL SYSTEM DESIGN FOR RIGID ROTOR

### Rigid rotor modeling

In this study, the control object is five axes control type magnetic bearing which is shown in FIGURE 1, but we can consider that is four-degree-of freedom when the

axial direction was assumed to be what is controlled ideally. Moreover, the unbalance and gyroscope effect was not considered.

FIGURE 2 shows the center of gravity position and the rotation angle around the x and the y axis are made  $x$ ,  $y$ ,  $\theta_y$  and  $\theta_x$ . We assume the  $\theta_y$  and  $\theta_x$  are very small here, then the gap between the upper and lower electromagnet and the rotor can be written:  $x_u = x + \theta_y l_u$ ,  $y_u = y + \theta_x l_u$ ,  $x_l = x - \theta_y l_l$  and  $y_l = y - \theta_x l_l$ . We can obtain the independent parallel and rotational equations of motion are shown follows each other, and TABLE 1 shows the those parameters.

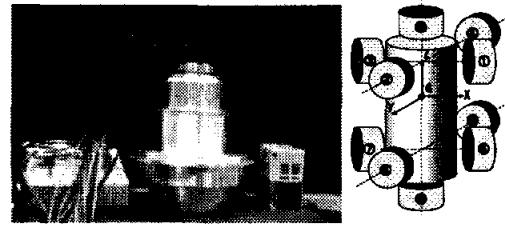


FIGURE 1: Five-DOF magnetic bearing system and model

$$M\ddot{x} = k_u \frac{i_{p1}^2}{(X_0 - x_u)^2} - k_u \frac{i_{p3}^2}{(X_0 + x_u)^2} + k_l \frac{i_{p5}^2}{(X_0 - x_l)^2} - k_l \frac{i_{p7}^2}{(X_0 + x_l)^2} \quad (1)$$

$$M\ddot{y} = k_u \frac{i_{p2}^2}{(Y_0 - y_u)^2} - k_u \frac{i_{p4}^2}{(Y_0 + y_u)^2} + k_l \frac{i_{p6}^2}{(Y_0 - y_l)^2} - k_l \frac{i_{p8}^2}{(Y_0 + y_l)^2} \quad (2)$$

$$I_r \ddot{\theta}_x = -k_u l_u \frac{i_{r2}^2}{(Y_0 - y_u)^2} + k_u l_u \frac{i_{r4}^2}{(Y_0 + y_u)^2} + k_l l_l \frac{i_{r6}^2}{(Y_0 - y_l)^2} - k_l l_l \frac{i_{r8}^2}{(Y_0 + y_l)^2} \quad (3)$$

$$I_r \ddot{\theta}_y = k_u l_u \frac{i_{r1}^2}{(X_0 - x_u)^2} - k_u l_u \frac{i_{r3}^2}{(X_0 + x_u)^2} - k_l l_l \frac{i_{r5}^2}{(X_0 - x_l)^2} + k_l l_l \frac{i_{r7}^2}{(X_0 + x_l)^2} \quad (4)$$

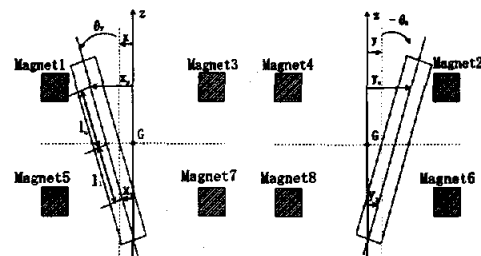


FIGURE 2: Parallel and rotational motion

### Control system design on rigid mode

From assumption, since the gyroscope effect is not

taken into consideration, the four equations (1)~(4) are mutually independent, and it can be handled as a thing without a couple. The new method which is related to the control input calculation was proposed for the subsystem such as parallel and rotational motion of the  $x$  and the  $y$  axis direction respectively. In this study, we designed the control system for the equations (1)~(4) simultaneously based on the backstepping method.

TABLE 1: System parameters and values

M	Mass of the rotor	4.85kg
$k_u$ $k_l$	Constant of magnetic bearing attractive force	$4.47 \times 10^{-6} \text{ Nm}^2 / \text{A}^2$ $3.10 \times 10^{-5} \text{ Nm}^2 / \text{A}^2$
$X_0, Y_0$	Nominal air gap	$2.5 \times 10^{-4} \text{ m}$
$I_r$	Moment of inertia about x and y axis	$0.029 \text{ kgm}^2$
$I_z$	Moment of inertia about z axis	$0.022 \text{ kgm}^2$
$l_u$ $l_l$	Distance from the center of gravity	$4.166 \times 10^{-2} \text{ m}$ $7.602 \times 10^{-2} \text{ m}$

Here, we define the vector as  $X=[x, y, \theta, \theta_x]^T$  and  $X_2 = \dot{X}_1$ , we obtain the eight orders system of the state space model from equations (1)~(4) was shown as follows:

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = U \end{cases} \quad (5)$$

where,  $U=[U_{xp} \ U_{yr} \ U_{yp} \ U_{xr}]^T$ .

$$\begin{cases} U_{xp} = U_{p1} - U_{p3} + U_{p5} - U_{p7} \\ U_{yr} = U_{r1} - U_{r3} - U_{r5} + U_{r7} \\ U_{yp} = U_{p2} - U_{p4} + U_{p6} - U_{p8} \\ U_{xr} = -U_{r2} + U_{r4} + U_{r6} - U_{r8} \end{cases} \quad (6)$$

Here,

$$\begin{cases} U_{pm} = k_{up} a_m i_{pm}^2 \\ U_{pn} = k_{ip} a_n i_{pn}^2 \quad (m=1,2,3,4, n=5,6,7,8) \\ U_{rm} = k_{ur} a_m i_{rm}^2 \\ U_{rn} = k_{ir} a_n i_{rn}^2 \end{cases} \quad (7)$$

where,

$$\begin{aligned} a_1 &= (X_0 - x_u)^{-2}, a_2 = (Y_0 - y_u)^{-2}, a_3 = (X_0 + x_u)^{-2}, a_4 = (Y_0 + y_u)^{-2}, \\ a_5 &= (X_0 - x_l)^{-2}, a_6 = (Y_0 - y_l)^{-2}, a_7 = (X_0 + x_l)^{-2}, a_8 = (Y_0 + y_l)^{-2}. \\ k_{up} &= \frac{k_u}{M}, k_{ip} = \frac{k_l}{M}, k_{ur} = \frac{k_u l_u}{I_r}, k_{ir} = \frac{k_l l_l}{I_r}, \end{aligned}$$

$X_2$  is considered as the virtual input. The control law will be shown as follows:

$$X_2 = -G_1 X_1 \quad (8)$$

to become  $X_{2ref} = -G_1 X_1$  ( $G_1$  is diagonal and regular).

We assume the control Lyapunov function which is

$$V_1 = X_1^T P X_1, \quad \dot{V}_1 \text{ will be shown as follows:}$$

$$\dot{V}_1 = -(X_1^T G_1 P X_1)^T - X_1^T G_1 P X_1 \quad (9)$$

Then Eq.(8) will be asymptotic stability. We assume the deviation  $Z_1$  was written as follows:

$$Z_1 = X_2 + G_1 X_1 \quad (10)$$

In the case, we define a new Lyapunov function including the deviation  $Z_1$  which is  $V_2 = X_1^T P X_1 + Z_1^T Q Z_1$  ( $P$  and  $Q$  was the diagonal and not singular).  $\dot{V}_2$  will be shown as follows:

$$\dot{V}_2 = -(X_1^T G_1 P X_1)^T - X_1^T G_1 P X_1 + Z_1^T Q (Q^{-1} P X_1 + U + G_1 X_2)^T + (Q^{-1} P X_1 + U + G_1 X_2)^T Q Z_1 \quad (11)$$

Here, we suppose that  $Q^{-1} P X_1 + U + G_1 X_2 = -G_2 Z_1$ , then

$\dot{V}_2$  will become:

$$\begin{aligned} \dot{V}_2 &= -(X_1^T G_1 P X_1)^T - X_1^T G_1 P X_1 \\ &\quad - (Z_1^T Q G_2 Z_1)^T - Z_1^T Q G_2 Z_1 \end{aligned} \quad (12)$$

It can confirm that the negative condition is satisfied. Then the virtual input of the system was obtained as follows:

$$U = P X_1 Q^{-1} - G_1 G_2 X_1 - (G_1 + G_2) X_2 \quad (13)$$

In this case, the input currents of the parallel and rotational direction for the electromagnets 1,3,5,7 which in the  $X$  axis was determined as follows by the switching method.

$$\begin{cases} U_{xp} \geq 0; U_{p3} = U_{p7} = 0; i_{p3} = i_{p7} = 0; \\ U_{p1} = U_{p5} = \frac{1}{2} U_{xp}; i_{p1}^2 = \frac{U_{p1}}{k_{up} a_1}; i_{p5}^2 = \frac{U_{p5}}{k_{ip} a_5} \end{cases}$$

$$\begin{cases} U_{xp} < 0; U_{p1} = U_{p5} = 0; i_{p1} = i_{p5} = 0; \\ U_{p3} = U_{p7} = \frac{1}{2} U_{xp}; i_{p3}^2 = \frac{U_{p3}}{k_{up} a_3}; i_{p7}^2 = \frac{U_{p7}}{k_{ip} a_7} \end{cases}$$

$$\begin{cases} U_{ry} \geq 0; U_{r3} = U_{r5} = 0; i_{r3} = i_{r5} = 0; \\ U_{r1} = U_{r7} = \frac{1}{2} U_{ry}; i_{r1}^2 = \frac{U_{r1}}{k_{ur} a_1}; i_{r7}^2 = \frac{U_{r7}}{k_{ir} a_7} \end{cases}$$

$$\begin{cases} U_{ry} < 0; U_{r1} = U_{r7} = 0; i_{r1} = i_{r7} = 0; \\ U_{r3} = U_{r5} = \frac{1}{2} U_{ry}; i_{r3}^2 = \frac{U_{r3}}{k_{ur} a_3}; i_{r5}^2 = \frac{U_{r5}}{k_{ir} a_5} \end{cases}$$

We can find out the control currents with the same process toward the electromagnets 2,4,6,8 which in the  $Y$  axis, it are omitted here. Finally, put the control currents of the parallel and rotational

direction the control currents was found out to each electromagnets was shown as follows:

$$i_j = \sqrt{i_{pj}^2 + i_{rj}^2}, (j=1,2,\dots,8) \quad (14)$$

The method which is found out the control input currents by that the plus/minus of the virtual input was judged, so the complex judgment condition by the plus/minus of the displacement and the velocity becomes unnecessary, and the problem which the imaginary number exist in the root inside can be avoided completely.

### Simulation (1)

In this section, the designed controller was applied to the four degree-of-freedom model. The simulation results of only the  $X$  direction was shown in FIGURE 3 and FIGURE 4. The results of the  $Y$  direction was omitted. In the case, the each parameters was designed respectively as follows:  
 $G_1 = \text{diag}[600,600,600,600], G_2 = \text{diag}[500,500,500,500]$ .  
 We obtain a good results without the over short from this simulation of the step and the impulse response. Furthermore, the zero power type control is realizable as such as that the input currents becomes zero when the rotor is stabilized and decoupled.

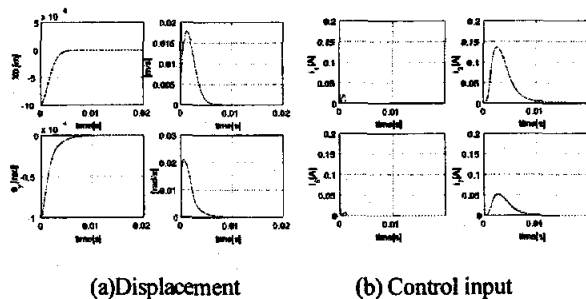


FIGURE 3: Step response

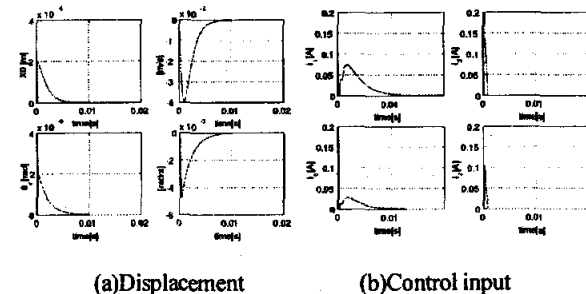


FIGURE 4: Impulse response

### Experimental results on rigid mode

In this section, we installed the designed controller in DSP to do experiment. The block diagram of the control system is shown in FIGURE 5.

In this experiment, we examines the levitation response from touchdown response, and the results was shown in FIGURE 6(a). In this case, the control currents is shown in FIGURE 6(b). It shows that the average value of the input currents are supplied on the electromagnets as 0.17A, 0.18A, 0.21A and 0.25A respectively. It shows the validity of the algorithm proposed in Fig.6 from that the rotor was stabilized and decoupled after control start within 0.03s.

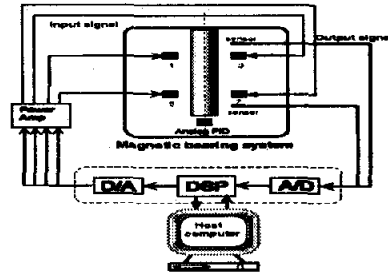


FIGURE 5: Experimental set up

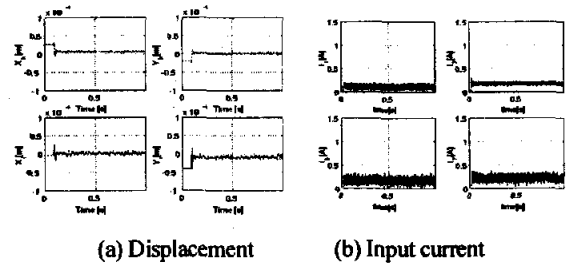


FIGURE 6: Step response

Furthermore, FIGURE 7(a) shows the orbits of the rotor about the experiment data in the case of 8000rpm, also, FIGURE 7(b) shows the input currents are 0.14 ~ 0.18A in that case. It is corresponded when the rotor is rotating and has being decoupled. In the case which the rotor was decoupled stable, the steady state error exists, namely that because of the disturbance exists in the actual rotor system.

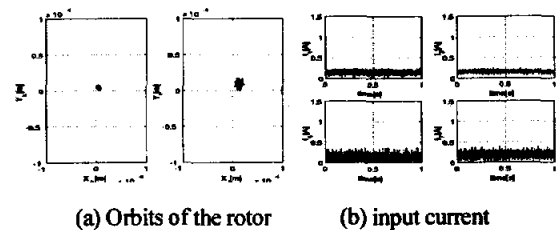


FIGURE 7: Rotation at 8000rpm

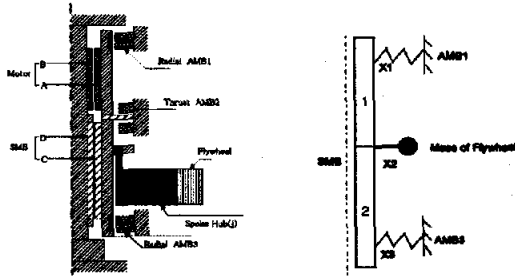
## CONTROL SYSTEM DESIGN ON FLEXIBLE ROTOR

### Flexible rotor modeling<sup>(9)</sup>

In this section, the control system design method for rigid rotor is extended to flexible rotor. As example,

we consider the model of the 10MWh class energy storage flywheel system, and design the control system based on the backstepping method. The structure and the one dimensional finite element mode(FEM) is shown in FIGURE 8 (a). The weight of this flywheel system is 104t, the weight of the rotor is 45.7t, and the operational speed is 6000rpm. The superconductivity magnet is applied to the levitation of the flywheel system, but also applied to the both axial and radial direction. The characteristic frequency of the 1 dimensional bending mode was decided which is 32Hz using the vibration analysis software ANSYS.

It is necessary to conduct the low dimensional finite element model from three dimensional model to order design the control system for AMB using ANSYS. In this case, the dimensional model is created making in agreement main parameters, such as total weight, entire length and characteristic frequency. After that, the model was divided into two parts based on the position and stiffness of AMB and SMB in order to prepare the one dimensional element model is shown in FIGURE 8 (b).



(a) Cross-sectional view (b) One dimensional FEM

FIGURE 8: Outer rotor type flywheel

Next, we derive the state equation of the flywheel based on FIGURE 8 (b). It is assumed that the states of the X and the Y directions are the same, so we only consider the X direction here. In the case, we applied the finite element method to the free-free flexible rotor, then the equation of motion was obtained as follows:

$$M\ddot{q} + C\dot{q} + Kq = 0 \quad (15)$$

However,  $q = [x_1, \theta_1, x_1, \theta_2, x_3, \theta_3]^T$ , the  $x_i, \theta_i$  is the displacement and the angle of the added mass respectively and  $x_1$  and  $x_3$  are the set place of the AMB as shown in FIGURE 8(b).

Here, we define  $\Phi$  is the modal matrix, and the

state equation of the flexible rotor-magnetic bearing system is obtained as follows:

$$\begin{aligned} \dot{x}_f &= A_f x_f + B_f U + D_f \omega \\ y &= C_f x_f = [x_1 \quad x_3]^T \end{aligned} \quad (16)$$

Here,  $x_f = [\xi \quad \xi]^T, U = [F_u \quad F_l]$

$$A_f = \begin{bmatrix} 0 & I \\ -\Omega^2 & -\Lambda \end{bmatrix}, B_f = \begin{bmatrix} 0 \\ f_l \end{bmatrix}, D_f = \begin{bmatrix} 0 \\ d \end{bmatrix}, C_f = [F^T \Phi \quad 0]$$

It is impossible to design the controller for the full order model because there are innumerable vibration mode in this free-free outer-rotor and including high dimensional bending mode because of the high-order controller. So the high vibration mode must be truncated to realize low order model. The state equation including the  $i^{\text{th}}$  dimensional was written as follows:

$$\begin{aligned} \dot{x}_r &= A_r x_r + B_r U + E_r \omega \\ y_r &= C_r x_r = [x_1 \quad x_3]^T \end{aligned} \quad (17)$$

In this study, since the two rigid modes and one bending mode exist within the operational speed by the ANSYS analysis, Eq.(17) includes those modes.

#### Control system design for flexible rotor

In this section, we only designed the control system for the low dimensional model of the X direction, and the equation of motion corresponding to the low dimensional model was written as follows:

$$M_m \ddot{X}_m + C_m \dot{X}_m + K_m X_m = B_m U \quad (18)$$

Here,  $X_m = [x_{m1}, x_{m2}, x_{m3}]^T$ .  $U = [F_u \quad F_l]^T$ , is the inputs of the AMB1 and AMB3. In this case

$$\begin{aligned} F_u &= k_u \frac{i_1^2}{(X_0 - x_1)^2} - k \cdot \frac{i_2^2}{(X_0 + x_1)^2} \\ F_l &= k_u \frac{i_3^2}{(X_0 - x_3)^2} - k \cdot \frac{i_4^2}{(X_0 + x_3)^2} \end{aligned} \quad (19)$$

However,  $k_u, k_l$  are the spring coefficients of AMB1 and AMB3. Here, we can obtain the equation as follows:

$$\begin{aligned} X_{m1} &= X_m \\ X_{m2} &= \dot{X}_{m1} \\ \dot{X}_{m2} &= -C_{m0} \dot{X}_{m1} - K_{m0} X_{m1} + B_{m0} U \end{aligned} \quad (20)$$

Where,  $C_{m0} = \frac{C_m}{M_m}, K_{m0} = \frac{K_m}{M_m}, B_{m0} = \frac{B_m}{M_m}$ .

Here, we assume the stabilizing function as  $\alpha(X_{m1}) = -G_1 X_{m1}$ , ( $G_1$  is diagonal and not singular),

then the error function is obtained as follows:

$$Z_1 = X_{m2} - \alpha(X_{m1}) = X_{m2} + G_1 X_{m1} \quad (21)$$

Then, the Lyapunov function was chosen as follows:

$$V = X_{m1}^T P X_{m1} + Z_1^T Q Z_1 \quad (22)$$

However,  $P, Q$  are diagonal and not singular.

$$\begin{aligned} \dot{V}_2 &= \dot{X}_{m1}^T P X_{m1} - X_{m1}^T P \dot{X}_{m1} + \dot{Z}_1^T Q Z_1 + Z_1^T Q \dot{Z}_1 \\ &= -X_{m1}^T P G_1 X_{m1} - (X_{m1}^T P G_1 X_{m1})^T \\ &\quad + Z_1^T Q (Q^{-1} P^T X_{m1} + B_{m0} U - C_{m0} X_{m2} - K_{m0} X_{m1})^T \\ &\quad + (Q^{-1} P^T X_{m1} + B_{m0} U - C_{m0} X_{m2} - K_{m0} X_{m1})^T Q Z_1 \end{aligned} \quad (23)$$

We propose the following condition:

$$Q^{-1} P^T X_{m1} + B_{m0} U - C_{m0} X_{m2} - K_{m0} X_{m1} = G_2 Z_1 \quad (24)$$

Here,  $G_2$  is diagonal and not singular. then Eq.(23) can be rewritten as follows:

$$\dot{V}_2 = -X_{m1}^T P G_1 X_{m1} - (X_{m1}^T P G_1 X_{m1})^T - Z_1^T Q G_2 Z_1 - (Z_1^T Q G_2 Z_1)^T \quad (25)$$

Because Eq.(25) becomes negative definite, the stability of the system was guaranteed. In this case, the virtual input was decided immediately as follows:

$$U = -B_{m0}^{-1} [(G_1 + G_2 - C_{m0}) X_{m2} + (G_1 G_2 + Q^{-1} P^T - K_{m0}) X_{m1}] \quad (26)$$

Here, we used the same process which with above chapter and to find out the input currents to the AMB1 and the AMB3 based on that the plus/minus of the virtual input was assumed. For the two upper electromagnets:

$$\begin{cases} F_u \geq 0; i_3 = 0; i_1 = (X_0 - x_1) \sqrt{\frac{F_u}{k_u}} \\ F_u < 0; i_1 = 0; i_3 = (X_0 + x_1) \sqrt{\frac{F_u}{k_u}} \end{cases} \quad (27)$$

For the lower two electromagnets:

$$\begin{cases} F_l \geq 0; i_7 = 0; i_5 = (X_0 - x_3) \sqrt{\frac{F_l}{k_l}} \\ F_l < 0; i_5 = 0; i_7 = (X_0 + x_3) \sqrt{\frac{F_l}{k_l}} \end{cases} \quad (28)$$

The problem which the control input become an imaginary number can be avoid completely using the above switching law.

### Servo system design

It is shown that the state disturbance was existed in actual control system from the results which show in FIGURE 7. So, in this study, we also design the servo system to control it based on the backstopping method. FIGURE 9 shows that the  $r, e, X_r$  were assumed such as the desired value, the error between

the desired value and the output, and the integration of that error respectively, the disturbance was assumed as step disturbance. Here, we also only design the servo system on the  $X$  axis direction for parallel motion.

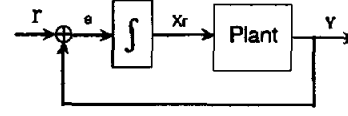


FIGURE 9: Servo system

$$\begin{cases} X_r = e \\ \dot{e} = \dot{r} - \dot{X} \\ \dot{X} = U \end{cases} \quad (29)$$

Here, that the desired value  $r = 0$ , the output is  $X$  and

$X_1 = X_r, X_2 = X, X_3 = \dot{X}$ , then we obtained the

equation from Eq.(29) as follows:

$$\begin{cases} \dot{X}_1 = X_2 \\ X_2 = X_3 \\ X_3 = U \end{cases} \quad (30)$$

In this case, we can find out the virtual inputs  $U$  after all the states of Eq.(30) would be changed as follows:

$$\begin{aligned} U &= -B_{m0}^{-1} [(G_1 + G_1 G_2 G_3 + G_3) X_{m1} \\ &\quad + (G_1 + G_2 + G_3 - C_{m0}) X_{m2} \\ &\quad + (G_1 G_2 + G_2 G_3 + G_1 G_3 + 2Q^{-1} P^T - K_{m0}) X_{m1}] \end{aligned} \quad (31)$$

Next, we can find out the input currents for each AMB by Eqs.(27) and (28).

### Simulation (2)

The control system was designed in the above section was applied to the flywheel system including two rigid modes and one bending mode by simulation. In this case, the low pass filter was used to avoid a spillover in the high frequency mode. The cut off frequency of the low pass filter was set up at 70Hz. Furthermore, we also consider the suitable damping term ( $\zeta=0.01$ ) for modeling error to stabilize.  $P, Q$  are unit matrix, and designed the parameters as:

$$G_1 = \text{diag}[100, 2, 15], G_2 = \text{diag}[150, 3000, 200],$$

FIGURE 10 shows the initial responses when the spring coefficient of SMB is given as  $K_{SMB}=1.064 \times 10^7$  N/m, and FIGURE 11 shows the initial responses when  $K_{SMB} = 0$  N/m. The control currents were decided by the switching type.

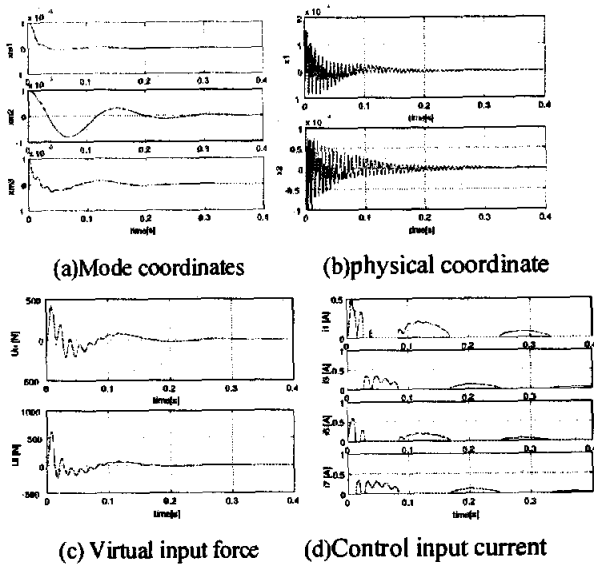


FIGURE 10: Initial value response ( $K_{SMB}=1.064 \times 10^7$  N/m)

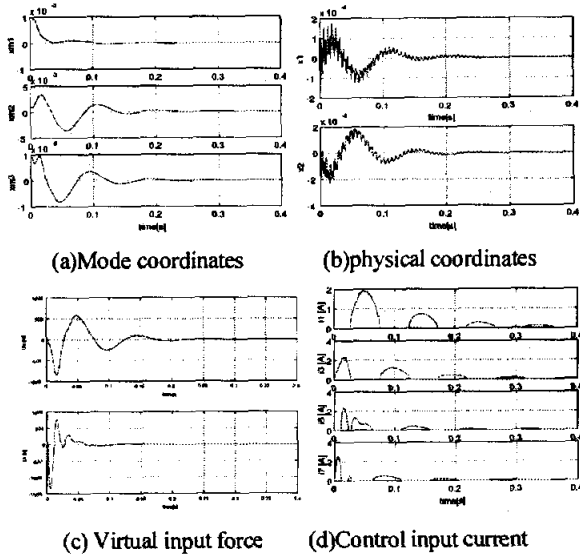


FIGURE 11: Initial value response ( $K_{SMB}=0$  N/m)

## CONCLUSIONS

In this study, the new zero-power nonlinear control algorithm was proposed based on the backstepping method for a magnetic bearing which it is only consists of a couple of electromagnets. Furthermore, we also propose a new method to find out the control input current to electromagnet to decide the plus/minus of the virtual input, and the problem which the imaginary number exists in the root can be avoided completely. And then, we applied the proposed nonlinear control system algorithm to the rigid model by simulation, the result shows that the zero-power control which does not supply a bias current is realizable and an improvement of the power consumption becomes possible drastically. The results of the experiment also confirm the validity of the proposed algorithm. However, since

the disturbance exist in the actual rotor system, it is impossible to disregard it in the experimental result, but it can be ignored because it no influences for the control performance from the result when rotating with high speed. And furthermore, we also examined the application possibility which applied the proposed method to the model of the 10MWh class energy storage flywheel system as such as flexible rotor. The results shows that the proposed algorithm can apply to the flexible rotor system, and the validity was verified.

## References

- (1) K.Nonami, K.Nishina, M.Saito, Discrete Time Sliding Mode Control of Zero Power Magnetic Bearing System. *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol.62, No.595, C(1996), p.967-975.
- (2) Charara,A., Miras,J.D. and Caron,B., Nonlinear Control of a Magnetic Bearing Levitation System Without Premagnetization, *IEEE, Trans. control Systems Tech.*, 4-5 (1996), 513-523.
- (3) Levine, J., Lottin, J. and Posart, J.C., A Nonlinear Approach to the Control of Magnetic Bearing, *IEEE Trans. Control Systems Tech.*, 4-5 (1996), 524-544.
- (4) T.Torres, M., Ramirez, H.S. and Escobar,G., Sliding Mode Nonlinear Control of Magnetic Bearings. *IEEE Int. Conf Control Applications*, (1999),743-748.
- (5) Y.Ariga, K.Nonami, and K.Sakai, Nonlinear Control of Zero Power Magnetic Bearing System (1st Report , Proposal of Control Algorithm Based on Lyapunov's Direct Method), *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol.67, No.654, C(2001), p.392-400.
- (6) Y.Ariga, K.Nonami, and K.Sakai, Nonlinear Control of Zero Power Magnetic Bearing (2nd Report , Experimental Consideration), *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol.67, No.654, C(2001), p.401-406.
- (7) K.Sakai, K.Nonami, and Y.Ariga, Zero Power Nonlinear Control of Magnetic Bearing System by means of Backstepping procedure, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), This paper is being contributed.
- (8) M.Krstić, I.Kanelakopoulos, P.Kokotović, Nonlinear and adaptive control design, Wiley, 1995.
- (9) Y.J.Zhang, K.Nonami, H.Higasa, and Z.H.Liu, Feasibility Study of Modeling and Control for 10MWh Class Energy Storage Flywheel System Using Superconducting Magnetic Bearing, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), This paper will be published.

