AN ANALYSIS OF AN INDUCTION BEARINGLESS MOTOR WITH A SQUIRREL CAGE ROTOR

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ABSTRACT

In induction bearingless motors, a squirrel cage rotor induces rotor currents by both corresponding motor and suspension magnetic field. The induced current causes a delay and loss in suspension force generation. Therefore, it is necessary to consider the rotor current in a squirrel cage rotor.

In this paper, radial force equation of bearingless induction motors with a squirrel cage rotor is derived in the detail. In both a rotor and a stator, 4-pole and 2-pole windings in three phases are assumed. A basic relation between flux linkages and winding currents is written by a 12 × 12 inductance matrix. Then, the matrix is transformed into an 8 × 8 inductance matrix. Radial forces are derived from a stored magnetic energy. Further assumption of symmetrical currents with a singular frequency is considered. Radial forces are found to be a product of airgap flux component currents.

INTRODUCTION

In induction bearingless motors, there are 4-pole and 2-pole winding sets in a stator. Thus, two types of bearingless motors are possible, one has 2-pole motor windings and the other has 4-pole motor windings. In 4-pole motor type, 4-pole-specific short circuits are sometimes necessary in a rotor to realize a better radial force response [1-2]. In 2-pole motor type, 2-pole-specific short circuits in a rotor also provide better response, however, a squirrel cage short circuit is employed in some cases [3-5]. There is an advantage in a squirrel cage rotor such as a simple structure, easy fabrication and better mechanical robustness. In a squirrel cage short circuit, both 4-pole and 2-pole magnetic field induces 4-pole and 2-pole currents. The current induction causes a response delay in radial force with respect to suspension winding current. Thus, the induced rotor currents have to be considered in the analysis. Rotor and stator currents both in 4-pole and 2-pole windings are totally 12 variables for three-phase winding system. Thus, an inductance matrix of 12 × 12 is derived. To the author's best knowledge, the analysis has not published in the literature. The 12 × 12 matrix is composed of 6 × 6 general induction motor matrix for 2-pole and 4-pole windings, and a unique mutual inductance matrix between 4-pole and 2-pole windings. The matrix is transformed into 2-phase system to obtain an 8 × 8 matrix. Radial forces are derived from the 8 × 8 matrix. Further assumptions of symmetrical currents with a singular frequency are introduced to obtain practically important consideration.

ARRANGEMENT OF STATOR WINDINGS

Fig.1 shows a cross section of a stator core, and an arrangement of stator windings. Conductors N_{2u} are connected in series to have 2-pole MMF distribution. The conductors N_{2v} and N_{2w} are also connected in the similar way to realize one set of three-phase windings. Another set of three-phase windings is composed of N_{4u}, N_{4v}, and N_{4w} conductors, which are enclosed in the stator slots.

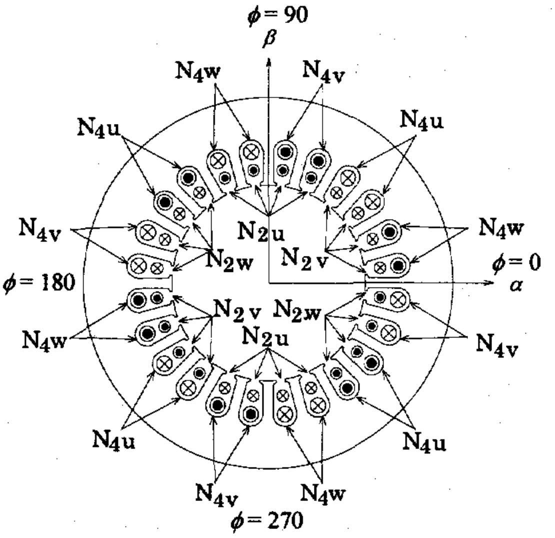


FIGURE 1: Arrangement of stator windings.

EQUIVALENT CIRCUITS

Figs. 2 show equivalent circuits of a squirrel cage induction type bearingless motor with the stator winding arrangement shown in Fig.1. As the stator has 4-pole and 2-pole winding sets, both 4-pole and 2-pole currents are induced in squirrel cage windings. Thus equivalent circuits are drawn separately for 2-pole and 4-pole circuits. Figs.2 (a) and (b) show 2-pole and 4-pole equivalent circuits, respectively. Three-phase windings **u,v,w** and **u',v',w'** are drawn in the stator core. Three-phase ac currents i_{us}, i_{vs} and i_{ws} with an angular frequency ω_2 flow into the 2-pole stator windings, on the contrary, three-phase ac currents $i_{u's}$, $i_{v's}$ and $i_{w's}$ with on angular frequency ω_4 flow into the 4-pole stator windings. Three-phase 2-pole rotor windings are assumed to carry currents i_{ur}, i_{vr} and i_{wr} having a slip angular frequency ω_{2s}. Moreover, three-phase 4-pole rotor windings are similarly assumed in the rotor, with currents $i_{u'r}$, $i_{v'r}$ and $i_{w'r}$ having a slip angular frequency ω_{4s}. The stator phase voltages from the neutral points to the terminals are defined as v_{us}, v_{vs}, v_{ws}, $v_{u's}$, $v_{v's}$ and $v_{w's}$. The rotor is at a rotational mechanical angular position of θ_{rm} in the counter clockwise direction with a shaft angular speed ω_{rm}. The 4-pole windings in Fig.2 (b) is drawn for the electrical angle of 360 deg, that is 180 deg in mechanical angle. Winding resistances and inductances are defined as follows;

R_{2s}, R_{4s}: Stator resistances
l_{2s}, l_{2r}, l_{4s}, l_{4r}: Leakage inductances of a stator and a rotor
M_2^{uvw} ,M_4^{uvw} : Effective inductances
R_{2r}, R_{4r}: Rotor resistances
L_{2s}^{uvw}, L_{4s}^{uvw}: Self-inductances of stator windings
L_{2r}^{uvw}, L_{4r}^{uvw}: Self-inductances of rotor windings

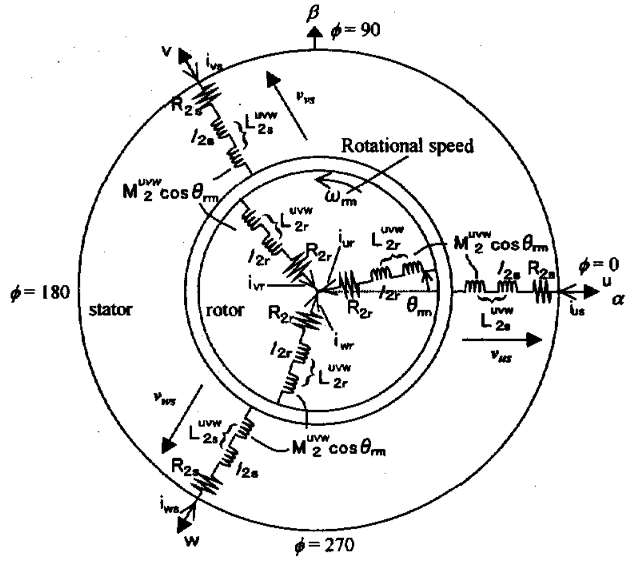
Machine parameters of the 2-pole and 4-pole windings are basically distinguished by the subscripts of 2 and 4. However, the dash is used in voltage and current of the 4-pole windings to have simple expressions. The L_{2s}^{uvw} , L_{2r}^{uvw} , L_{4s}^{uvw} and L_{4r}^{uvw} are the sums of the leakage inductance and each effective inductance, as,

$$\begin{aligned} L_{2s}^{uvw} &= l_{2s} + M_2^{uvw} \\ L_{2r}^{uvw} &= l_{2r} + M_2^{uvw} \\ L_{4s}^{uvw} &= l_{4s} + M_4^{uvw} \\ L_{4r}^{uvw} &= l_{4r} + M_4^{uvw} \end{aligned} \tag{1}$$

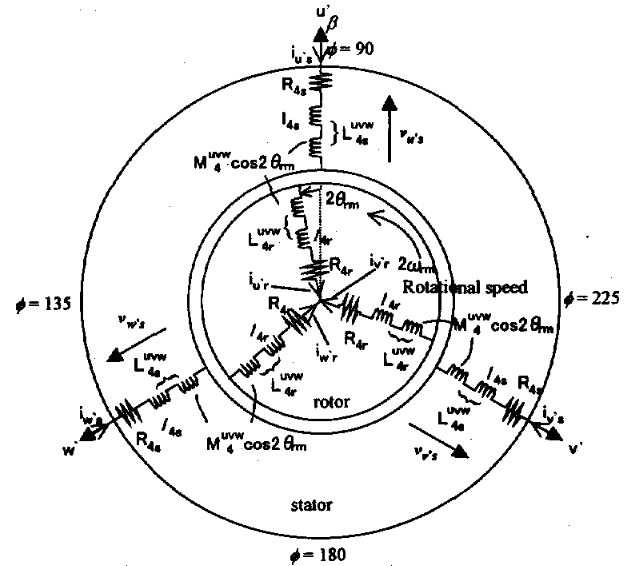
Mutual inductance between stator and rotor windings are a product of an effective inductance and a cosine function of a mechanical angular position θ_{rm}.

INDUCTANCE MATRIX

Let us define that flux linkages corresponding to each winding are λ_{us}, λ_{vs}, λ_{ws}, λ_{ur}, λ_{vr}, λ_{wr}, $\lambda_{u's}$, $\lambda_{v's}$, $\lambda_{w's}$, $\lambda_{u'r}$, $\lambda_{v'r}$ and $\lambda_{w'r}$. A relationship between flux linkages and currents are expressed by an inductance matrix of 12 rows and 12 columns. Let us define that $[\lambda_{uvwu'v'w'}]$ and $[i_{uvwu'v'w'}]$ are flux



(a) 2-pole windings.



(b) 4-pole windings.

FIGURES 2: Equivalent circuits of an induction type bearingless motor with a squirrel cage rotor.

linkage and current vectors. Let us also define that the 12×12 matrix as $[L_{uvwu'v'w'}]$, then,

$$[\lambda_{uvwu'v'w'}] = [L_{uvwu'v'w'}][i_{uvwu'v'w'}] \tag{2}$$

where,

$$[\lambda_{uvwu'v'w'}] = [\lambda_{us}\ \lambda_{vs}\ \lambda_{ws}\ \lambda_{ur}\ \lambda_{vr}\ \lambda_{wr}\ \lambda_{u's}\ \lambda_{v's}\ \lambda_{w's}\ \lambda_{u'r}\ \lambda_{v'r}\ \lambda_{w'r}]^t$$

$$[i_{uvwu'v'w'}] = [i_{us}\ i_{vs}\ i_{ws}\ i_{ur}\ i_{vr}\ i_{wr}\ i_{u's}\ i_{v's}\ i_{w's}\ i_{u'r}\ i_{v'r}\ i_{w'r}]^t$$

$$[L_{uvwu'v'w'}] = \begin{bmatrix} [L_{11}^{uvw}] & [L_{12}^{uvw}] \\ [L_{21}^{uvw}] & [L_{22}^{uvw}] \end{bmatrix} \tag{3}$$

where, $[L_{11}^{uvw}]$, $[L_{12}^{uvw}]$, $[L_{21}^{uvw}]$ and $[L_{22}^{uvw}]$ are 6×6 matrices shown in (3a-3c). The matrices $[L_{11}^{uvw}]$ and $[L_{22}^{uvw}]$ are exactly the same as general inductance matrix of squirrel cage induction motors having 2-pole or 4-pole windings. On the contrary, $[L_{12}^{uvw}]$ and $[L_{21}^{uvw}]$ are unique to bearingless motors. The matrices $[L_{12}^{uvw}]$ and $[L_{21}^{uvw}]$ are mutual inductance matrices between 4-pole and 2-pole windings. If a rotor is located at a stator magnetic center, all elements are equal to zero. If the rotor radially shifts from the magnetic center, the flux distribution is unbalanced and mutual couplings are caused. These mutual inductances are expressed by rotor eccentric displacements α and β. The deviation of $[L_{12}^{uvw}]$ is described in the rest of this section.

Fig.3 shows airgap length, where, the rotor center o' is displaced in radial directions from the stator center o by α and β. The airgap length, when the rotor is centered, is defined as g_0. The rotational coordinate ϕ is defined as counter clockwise. It is assumed that eccentricity of the rotor is small enough to g_0. A radius of the rotor, an axial length of the core and the permeability in the air are defined as R, l

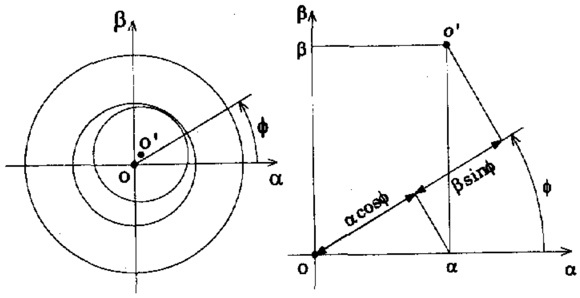


FIGURE 3: Displacement of rotor

and μ_0. A permeance P for one radian at ϕ is written as,

$$P=\frac{\mu_0 Rl}{g_0}\left(1+\frac{\alpha}{g_0}\cos\phi+\frac{\beta}{g_0}\sin\phi\right) \tag{4}$$

Let us assume that MMFs of 2-pole and 4-pole windings are distributed sinusoidally. From Fig.1, MMFs at unity current are written as follows,

$$[L_{11}^{uvw}]=\begin{bmatrix} L_{2s}^{uvw} & -\frac{M_2^{uvw}}{2} & -\frac{M_2^{uvw}}{2} & M_2^{uvw}\cos\theta_{rm} & M_2^{uvw}\cos(\theta_{rm}-\frac{4}{3}\pi) & M_2^{uvw}\cos(\theta_{rm}-\frac{2}{3}\pi) \\ -\frac{M_2^{uvw}}{2} & L_{2s}^{uvw} & -\frac{M_2^{uvw}}{2} & M_2^{uvw}\cos(\theta_{rm}-\frac{2}{3}\pi) & M_2^{uvw}\cos\theta_{rm} & M_2^{uvw}\cos(\theta_{rm}-\frac{4}{3}\pi) \\ -\frac{M_2^{uvw}}{2} & -\frac{M_2^{uvw}}{2} & L_{2s}^{uvw} & M_2^{uvw}\cos(\theta_{rm}-\frac{4}{3}\pi) & M_2^{uvw}\cos(\theta_{rm}-\frac{2}{3}\pi) & M_2^{uvw}\cos\theta_{rm} \\ M_2^{uvw}\cos\theta_{rm} & M_2^{uvw}\cos(\theta_{rm}-\frac{2}{3}\pi) & M_2^{uvw}\cos(\theta_{rm}-\frac{4}{3}\pi) & L_{2r}^{uvw} & -\frac{M_2^{uvw}}{2} & -\frac{M_2^{uvw}}{2} \\ M_2^{uvw}\cos(\theta_{rm}-\frac{4}{3}\pi) & M_2^{uvw}\cos\theta_{rm} & M_2^{uvw}\cos(\theta_{rm}-\frac{2}{3}\pi) & -\frac{M_2^{uvw}}{2} & L_{2r}^{uvw} & -\frac{M_2^{uvw}}{2} \\ M_2^{uvw}\cos(\theta_{rm}-\frac{2}{3}\pi) & M_2^{uvw}\cos(\theta_{rm}-\frac{4}{3}\pi) & M_2^{uvw}\cos\theta_{rm} & -\frac{M_2^{uvw}}{2} & -\frac{M_2^{uvw}}{2} & L_{2r}^{uvw} \end{bmatrix} \tag{3a}$$

$$[L_{22}^{uvw}]=\begin{bmatrix} L_{4s}^{uvw} & -\frac{M_4^{uvw}}{2} & -\frac{M_4^{uvw}}{2} & M_4^{uvw}\cos2\theta_{rm} & M_4^{uvw}\cos(2\theta_{rm}-\frac{2}{3}\pi) & M_4^{uvw}\cos(2\theta_{rm}-\frac{4}{3}\pi) \\ -\frac{M_4^{uvw}}{2} & L_{4s}^{uvw} & -\frac{M_4^{uvw}}{2} & M_4^{uvw}\cos(2\theta_{rm}-\frac{4}{3}\pi) & M_4^{uvw}\cos2\theta_{rm} & M_4^{uvw}\cos(2\theta_{rm}-\frac{2}{3}\pi) \\ -\frac{M_4^{uvw}}{2} & -\frac{M_4^{uvw}}{2} & L_{4s}^{uvw} & M_4^{uvw}\cos(2\theta_{rm}-\frac{2}{3}\pi) & M_4^{uvw}\cos(2\theta_{rm}-\frac{4}{3}\pi) & M_4^{uvw}\cos2\theta_{rm} \\ M_4^{uvw}\cos2\theta_{rm} & M_4^{uvw}\cos(2\theta_{rm}-\frac{4}{3}\pi) & M_4^{uvw}\cos(2\theta_{rm}-\frac{2}{3}\pi) & L_{4r}^{uvw} & -\frac{M_4^{uvw}}{2} & -\frac{M_4^{uvw}}{2} \\ M_4^{uvw}\cos(2\theta_{rm}-\frac{2}{3}\pi) & M_4^{uvw}\cos2\theta_{rm} & M_4^{uvw}\cos(2\theta_{rm}-\frac{4}{3}\pi) & -\frac{M_4^{uvw}}{2} & L_{4r}^{uvw} & -\frac{M_4^{uvw}}{2} \\ M_4^{uvw}\cos(2\theta_{rm}-\frac{4}{3}\pi) & M_4^{uvw}\cos(2\theta_{rm}-\frac{2}{3}\pi) & M_4^{uvw}\cos2\theta_{rm} & -\frac{M_4^{uvw}}{2} & -\frac{M_4^{uvw}}{2} & L_{4r}^{uvw} \end{bmatrix} \tag{3b}$$

$$[L_{12}^{uvw}]=[L_{21}^{uvw}]^t=-\frac{2}{3}M_{24}\left[\begin{array}{ccc|ccc} \alpha & -\frac{1}{2}\alpha-\frac{\sqrt{3}}{2}\beta & -\frac{1}{2}\alpha+\frac{\sqrt{3}}{2}\beta & \alpha\cos2\theta_{rm}+\beta\sin2\theta_{rm} & \alpha\cos(2\theta_{rm}-\frac{2\pi}{3})+\beta\sin(2\theta_{rm}-\frac{2\pi}{3}) & \alpha\cos(2\theta_{rm}-\frac{4\pi}{3})+\beta\sin(2\theta_{rm}-\frac{4\pi}{3}) \\ -\frac{1}{2}\alpha-\frac{\sqrt{3}}{2}\beta & -\frac{1}{2}\alpha+\frac{\sqrt{3}}{2}\beta & \alpha & \alpha\cos(2\theta_{rm}-\frac{2\pi}{3})+\beta\sin(2\theta_{rm}-\frac{2\pi}{3}) & \alpha\cos(2\theta_{rm}-\frac{4\pi}{3})+\beta\sin(2\theta_{rm}-\frac{4\pi}{3}) & \alpha\cos2\theta_{rm}+\beta\sin2\theta_{rm} \\ -\frac{1}{2}\alpha+\frac{\sqrt{3}}{2}\beta & \alpha & -\frac{1}{2}\alpha-\frac{\sqrt{3}}{2}\beta & \alpha\cos(2\theta_{rm}-\frac{4\pi}{3})+\beta\sin(2\theta_{rm}-\frac{4\pi}{3}) & \alpha\cos2\theta_{rm}+\beta\sin2\theta_{rm} & \alpha\cos(2\theta_{rm}-\frac{2\pi}{3})+\beta\sin(2\theta_{rm}-\frac{2\pi}{3}) \\ \hline \alpha\cos\theta_{rm}-\beta\sin\theta_{rm} & \alpha\cos(\theta_{rm}-\frac{4\pi}{3})-\beta\sin(\theta_{rm}-\frac{4\pi}{3}) & \alpha\cos(\theta_{rm}-\frac{2\pi}{3})-\beta\sin(\theta_{rm}-\frac{2\pi}{3}) & \alpha\cos\theta_{rm}+\beta\sin\theta_{rm} & \alpha\cos(\theta_{rm}-\frac{2\pi}{3})+\beta\sin(\theta_{rm}-\frac{2\pi}{3}) & \alpha\cos(\theta_{rm}-\frac{4\pi}{3})+\beta\sin(\theta_{rm}-\frac{4\pi}{3}) \\ \alpha\cos(\theta_{rm}-\frac{4\pi}{3})-\beta\sin(\theta_{rm}-\frac{4\pi}{3}) & \alpha\cos(\theta_{rm}-\frac{2\pi}{3})-\beta\sin(\theta_{rm}-\frac{2\pi}{3}) & \alpha\cos\theta_{rm}-\beta\sin\theta_{rm} & \alpha\cos(\theta_{rm}-\frac{2\pi}{3})+\beta\sin(\theta_{rm}-\frac{2\pi}{3}) & \alpha\cos(\theta_{rm}-\frac{4\pi}{3})+\beta\sin(\theta_{rm}-\frac{4\pi}{3}) & \alpha\cos\theta_{rm}+\beta\sin\theta_{rm} \\ \alpha\cos(\theta_{rm}-\frac{2\pi}{3})-\beta\sin(\theta_{rm}-\frac{2\pi}{3}) & \alpha\cos\theta_{rm}-\beta\sin\theta_{rm} & \alpha\cos(\theta_{rm}-\frac{4\pi}{3})-\beta\sin(\theta_{rm}-\frac{4\pi}{3}) & \alpha\cos(\theta_{rm}-\frac{4\pi}{3})+\beta\sin(\theta_{rm}-\frac{4\pi}{3}) & \alpha\cos\theta_{rm}+\beta\sin\theta_{rm} & \alpha\cos(\theta_{rm}-\frac{2\pi}{3})+\beta\sin(\theta_{rm}-\frac{2\pi}{3}) \end{array}\right] \tag{3c}$$

$$[A_2]=\begin{bmatrix}A_{us}\\A_{vs}\\A_{ws}\\A_{ur}\\A_{vr}\\A_{wr}\end{bmatrix}=N_{2a}\begin{bmatrix}\cos\phi\\\cos(\phi-\frac{2}{3}\pi)\\\cos(\phi-\frac{4}{3}\pi)\\\cos(\phi-\theta_{rm})\\\cos(\phi-\frac{2}{3}\pi-\theta_{rm})\\\cos(\phi-\frac{4}{3}\pi-\theta_{rm})\end{bmatrix} \quad (5)$$

$$[A_4]=\begin{bmatrix}A_{u's}\\A_{v's}\\A_{w's}\\A_{u'r}\\A_{v'r}\\A_{w'r}\end{bmatrix}=-N_{4a}\begin{bmatrix}\cos 2\phi\\\cos(2\phi-\frac{4}{3}\pi)\\\cos(2\phi-\frac{2}{3}\pi)\\\cos(2\phi-2\theta_{rm})\\\cos(2\phi-\frac{4}{3}\pi-2\theta_{rm})\\\cos(2\phi-\frac{2}{3}\pi-2\theta_{rm})\end{bmatrix} \quad (6)$$

where, N_{2a} and N_{4a} are amplitudes of a fundamental component. Positive directions of MMFs are defined in radial direction, thus, a negative sign exists in all elements of $[A_4]$. It is assumed that the number of turns of rotor windings is equal to that of stator windings.

Let us define magnetic potential of the rotor is V. Flux Ψ_{us} for one radian with only u-phase unit current excitation is given, as,

$$\Psi_{us}=P(\frac{1}{2}A_{us}+V) \quad (7)$$

Moreover, an integral of the flux around the rotor is 0, thus,

$$\int_0^{2\pi}\Psi_{us}d\phi=0 \quad (8)$$

Substituting Ψ_{us} in (7) and solving for V yields,

$$V=-\frac{\frac{1}{2}\int_0^{2\pi}PA_{us}d\phi}{\int_0^{2\pi}Pd\phi}=-\frac{N_{2a}}{4g_0}\alpha \quad (9)$$

Note that the rotor potential is only a function of a displacement α for u_s-winding excitation. The potentials V at other 2-pole winding excitation with a rotor eccentricity are similarly calculated. The potentials are a product of a displacement and a cosine function of an angle between the direction of MMF and the displacement. The 2-pole fluxes Ψ_{us}, Ψ_{vs}, Ψ_{ws}, Ψ_{ur}, Ψ_{vr} and Ψ_{wr} with only a corresponding 2-pole winding excitation are obtained as follows;

$$[\Psi_2]=\begin{bmatrix}\Psi_{us}\\\Psi_{vs}\\\Psi_{ws}\\\Psi_{ur}\\\Psi_{vr}\\\Psi_{wr}\end{bmatrix}=\frac{P}{2}\begin{bmatrix}A_{us}-\frac{N_{2a}}{2g_0}\alpha\\A_{vs}-\frac{N_{2a}}{2g_0}(-\frac{1}{2}\alpha+\frac{\sqrt{3}}{2}\beta)\\A_{ws}-\frac{N_{2a}}{2g_0}(-\frac{1}{2}\alpha-\frac{\sqrt{3}}{2}\beta)\\A_{ur}-\frac{N_{2a}}{2g_0}(\alpha\cos\theta_{rm}+\beta\sin\theta_{rm})\\A_{vr}-\frac{N_{2a}}{2g_0}\{(-\frac{1}{2}\alpha+\frac{\sqrt{3}}{2}\beta)\cos\theta_{rm}+(-\frac{\sqrt{3}}{2}\alpha-\frac{1}{2}\beta)\sin\theta_{rm}\}\\A_{wr}-\frac{N_{2a}}{2g_0}\{(-\frac{1}{2}\alpha-\frac{\sqrt{3}}{2}\beta)\cos\theta_{rm}+(\frac{\sqrt{3}}{2}\alpha-\frac{1}{2}\beta)\sin\theta_{rm}\}\end{bmatrix} \quad (10)$$

Mutual inductances can be derived by integrations the flux distribution in an equation (10) by multiplying the winding distribution given by (6). For instance, the mutual inductance $M_{usu's}$ between the 2-pole u-phase stator winding and the 4-pole u-phase stator winding is given as,

$$M_{usu's}=\int_0^{2\pi}\Psi_{us}\frac{A_{u's}}{2}d\phi \quad (11)$$

Equations (4-6,10) are substituted, as shown as follows,

$$M_{usu's}=\int_0^{2\pi}\Psi_{us}\frac{A_{u's}}{2}d\phi=\int_0^{2\pi}\frac{P}{2}(A_{us}-\frac{N_{2a}}{2g_0}\alpha)\frac{A_{u's}}{2}d\phi$$
$$=\int_0^{2\pi}(\frac{P}{4}A_{us}A_{u's}-\frac{PN_{2a}\alpha}{8g_0}A_{u's})d\phi$$
$$=\frac{1}{4}\int_0^{2\pi}\frac{\mu_0 Rl}{g_0}(1+\frac{\alpha}{g_0}\cos\phi+\frac{\beta}{g_0}\sin\phi)N_{2a}\cos\phi(-N_{4a})\cos 2\phi d\phi$$
$$-\frac{N_{2a}\alpha}{8g_0}\int_0^{2\pi}\frac{\mu_0 Rl}{g_0}(1+\frac{\alpha}{g_0}\cos\phi+\frac{\beta}{g_0}\sin\phi)(-N_{4a})\cos 2\phi d\phi$$

Executing the integration results in a simple equation as,

$$M_{usu's}=-\frac{\pi\mu_0 RlN_{2a}N_{4a}}{8g_0^2}\alpha \quad (12)$$

The equation (12) is (1,1) element of $[L_{12}^{uvw}]$ and $[L_{21}^{uvw}]$ in (3c). A constant M'_{24} is defined as,

$$M'_{24}=\frac{3\pi\mu_0 RlN_{2a}N_{4a}}{16g_0^2} \quad (13)$$

In the similar way, all elements are calculated as shown in (3c). Note that all elements in $[L_{12}^{uvw}]$ are a product of M'_{24}, radial rotor displacements, and triangular functions of θ_{rm}.

THREE－PHASE TO TWO－PHASE TRANSFORMATION

The derived inductance matrix $[L_{uvwu'v'w'}]$ is in a three-phase coordinates. To simplify the equation, three-phase to two-phase transformation is executed in this section. Three-phase to two-phase transformation matrix $[C_{32}]$ is given as,

$$[C_{32}]=\sqrt{\frac{2}{3}}\begin{bmatrix}1&-1/2&-1/2&0&0&0&0&0&0&0&0&0\\0&\sqrt{3}/2&-\sqrt{3}/2&0&0&0&0&0&0&0&0&0\\0&0&0&1&-1/2&-1/2&0&0&0&0&0&0\\0&0&0&0&\sqrt{3}/2&-\sqrt{3}/2&0&0&0&0&0&0\\0&0&0&0&0&0&1&-1/2&-1/2&0&0&0\\0&0&0&0&0&0&0&\sqrt{3}/2&-\sqrt{3}/2&0&0&0\\0&0&0&0&0&0&0&0&0&1&-1/2&-1/2\\0&0&0&0&0&0&0&0&0&0&\sqrt{3}/2&-\sqrt{3}/2\end{bmatrix} \quad (14)$$

The two-phase inductance matrix is 8×8 matrix. Let us define the two-phase inductance matrix as $[L_{\alpha\beta\alpha'\beta'}]$. $[L_{\alpha\beta\alpha'\beta'}]$ is given by,

$$[L_{\alpha\beta\alpha'\beta'}]=[C_{32}][L_{uvwu'v'w'}][C_{32}]^t \quad (15)$$

The calculated result of $[L_{\alpha\beta\alpha'\beta'}]$ is given in (16) and (16a-c).

$$[L_{\alpha\beta\alpha'\beta'}]=\begin{bmatrix}[L_{11}^{\alpha\beta}]&[L_{12}^{\alpha\beta}]\\[L_{21}^{\alpha\beta}]&[L_{22}^{\alpha\beta}]\end{bmatrix} \quad (16)$$

The following relations exist in two-phase inductances as,

$$
\begin{aligned}
M_2 &= \frac{3}{2} M_2^{uvw} & M_4 &= \frac{3}{2} M_4^{uvw} \\
L_{2s} &= l_{2s} + M_2 & L_{4s} &= l_{4s} + M_4 \\
L_{2r} &= l_{2r} + M_2 & L_{4r} &= l_{4r} + M_4
\end{aligned} \tag{17}
$$

Note that inductance matrix is simplified by the three-phase to two-phase transformation. The equation (16a) and (16b) are inductance matrices of 2-pole and 4-pole induction motors. The equation (16c) is original in bearingless motors. The two-phase current vector $[i_{\alpha\beta\alpha'\beta'}]$ are given as,

$$
[i_{\alpha\beta\alpha'\beta'}] = [C_{32}][i_{uvwu'v'w'}] \tag{18}
$$

where each element is given as follows.

$$
[i_{\alpha\beta\alpha'\beta'}] = [i_{\alpha s} \;\; i_{\beta s} \;\; i_{\alpha r} \;\; i_{\beta r} \;\; i_{\alpha' s} \;\; i_{\beta' s} \;\; i_{\alpha' r} \;\; i_{\beta' r}]^t \tag{19}
$$

RADIAL FORCE

It is necessary to calculate to derive the radial force. Let us assume that a magnetic circuit is linear, then magnetic energy W_m is written as,

$$
W_m = \frac{1}{2} [i_{\alpha\beta\alpha'\beta'}]^t \, [L_{\alpha\beta\alpha'\beta'}] \, [i_{\alpha\beta\alpha'\beta'}] \tag{20}
$$

Let us define that radial forces in α and β directions are F_α, and F_β, respectively. The radial forces are obtained by the partial derivatives with respect to the corresponding radial displacements as,

$$
\begin{bmatrix} F_\alpha \\ F_\beta \end{bmatrix} = \begin{bmatrix} \partial W_m / \partial \alpha \\ \partial W_m / \partial \beta \end{bmatrix} \tag{21}
$$

Substituting equations (16), (16a,b,c) and (19) into (20), then, calculating (21) yields,

$$
\begin{aligned}
F_\alpha = M'_{24} \{ & -i_{\alpha s} i_{\alpha' s} + i_{\beta s} i_{\beta' s} - \cos\theta_{rm} (i_{\alpha r} i_{\alpha' s} + i_{\alpha r} i_{\alpha' r} - i_{\beta r} i_{\beta' s} - i_{\beta r} i_{\beta' r}) \\
& - \sin\theta_{rm} (i_{\alpha r} i_{\beta' r} + i_{\beta r} i_{\alpha' r} - i_{\alpha r} i_{\beta' s} - i_{\beta r} i_{\alpha' s}) \\
& - \cos 2\theta_{rm} (i_{\alpha s} i_{\alpha' r} - i_{\beta s} i_{\beta' r}) - \sin 2\theta_{rm} (i_{\alpha s} i_{\beta' r} + i_{\beta s} i_{\alpha' r}) \}
\end{aligned} \tag{22a}
$$

$$
\begin{aligned}
F_\beta = M'_{24} \{ & i_{\alpha s} i_{\beta' s} + i_{\beta s} i_{\alpha' s} + \cos\theta_{rm} (i_{\alpha r} i_{\beta' s} + i_{\alpha r} i_{\beta' r} + i_{\beta r} i_{\alpha' s} + i_{\beta r} i_{\alpha' r}) \\
& + \sin\theta_{rm} (i_{\alpha r} i_{\alpha' s} + i_{\beta r} i_{\beta' r} - i_{\alpha r} i_{\alpha' r} - i_{\beta r} i_{\beta' s}) \\
& + \cos 2\theta_{rm} (i_{\alpha s} i_{\beta' r} + i_{\beta s} i_{\alpha' r}) + \sin 2\theta_{rm} (i_{\beta s} i_{\beta' r} - i_{\alpha s} i_{\alpha' r}) \}
\end{aligned} \tag{22b}
$$

These equations provide us general radial force expressions for arbitrary instantaneous currents in both stator and rotor windings. In order to obtain practical expressions, winding currents can be assumed as symmetrical triangular function with a singular frequency. The angular frequencies of 2-pole and 4-pole stator currents are ω_2 and ω_4. The corresponding rotor current frequencies are ω_{2s} and ω_{4s}, then,

$$
\left.
\begin{aligned}
&\begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix} = \begin{bmatrix} \cos\omega_2 t & -\sin\omega_2 t \\ \sin\omega_2 t & \cos\omega_2 t \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} \quad \text{(2-pole stator)} \\
&\begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \end{bmatrix} = \begin{bmatrix} \cos\omega_{2s} t & -\sin\omega_{2s} t \\ \sin\omega_{2s} t & \cos\omega_{2s} t \end{bmatrix} \begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix} \quad \text{(2-pole rotor)} \\
&\begin{bmatrix} i_{\alpha' s} \\ i_{\beta' s} \end{bmatrix} = \begin{bmatrix} \cos\omega_4 t & \sin\omega_4 t \\ -\sin\omega_4 t & \cos\omega_4 t \end{bmatrix} \begin{bmatrix} i_{d' s} \\ i_{q' s} \end{bmatrix} \quad \text{(4-pole stator)} \\
&\begin{bmatrix} i_{\alpha' r} \\ i_{\beta' r} \end{bmatrix} = \begin{bmatrix} \cos\omega_{4s} t & \sin\omega_{4s} t \\ -\sin\omega_{4s} t & \cos\omega_{4s} t \end{bmatrix} \begin{bmatrix} i_{d' r} \\ i_{q' r} \end{bmatrix} \quad \text{(4-pole rotor)}
\end{aligned}
\right\} \tag{23}
$$

The position of a negative sign of the sine functions is not the same between 4-pole and 2pole, because the phase sequence is opposite. The current amplitudes i_{ds}, i_{qs}, i_{dr}, i_{qr}, $i_{d's}$, $i_{q's}$, $i_{d'r}$ and $i_{q'r}$ are constant dc values in steady-state conditions. Let us assume that 2-pole and 4-pole slips are s_2 and s_4, respectively. These slips are given by,

$$
\begin{aligned}
s_2 &= \frac{\omega_2 - \omega_{rm}}{\omega_2} \\
s_4 &= \frac{\omega_4 - 2\omega_{rm}}{\omega_4}
\end{aligned} \tag{24}
$$

Therefore, the following expressions are obtained from the equation (24) considering that $\omega_{2s} = \omega_2 s_2$ and $\omega_{4s} = \omega_4 s_4$, as,

$$
\omega_2 = \omega_{rm} + \omega_{2s} \tag{25a}
$$

$$
\omega_4 = 2\omega_{rm} + \omega_{4s} \tag{25b}
$$

From (25a) and (25b), the following important relationship is derived.

$$
2\omega_2 - \omega_4 = 2\omega_{2s} - \omega_{4s} \tag{26}
$$

Equation (23) and (26) are substituted in to (22a,22b), considering that $\theta_{rm} = \omega_{rm} t$, then the calculation results in a simple expression as,

$$
[L_{11}^{\alpha\beta}] = \begin{bmatrix} L_{2s} & 0 & M_2 \cos\theta_{rm} & -M_2 \sin\theta_{rm} \\ 0 & L_{2s} & M_2 \sin\theta_{rm} & M_2 \cos\theta_{rm} \\ M_2 \cos\theta_{rm} & M_2 \sin\theta_{rm} & L_{2r} & 0 \\ -M_2 \sin\theta_{rm} & M_2 \cos\theta_{rm} & 0 & L_{2r} \end{bmatrix} \tag{16a}
$$

$$
[L_{22}^{\alpha\beta}] = \begin{bmatrix} L_{4s} & 0 & M_4 \cos 2\theta_{rm} & M_4 \sin 2\theta_{rm} \\ 0 & L_{4s} & -M_4 \sin 2\theta_{rm} & M_4 \cos 2\theta_{rm} \\ M_4 \cos 2\theta_{rm} & -M_4 \sin 2\theta_{rm} & L_{4r} & 0 \\ M_4 \sin 2\theta_{rm} & M_4 \cos 2\theta_{rm} & 0 & L_{4r} \end{bmatrix} \tag{16b}
$$

$$
[L_{12}^{\alpha\beta}] = [L_{21}^{\alpha\beta}]' = M'_{24} \left[\begin{array}{cc|cc} -\alpha & \beta & -\alpha\cos 2\theta_{rm} - \beta\sin 2\theta_{rm} & -\alpha\sin 2\theta_{rm} + \beta\cos 2\theta_{rm} \\ \beta & \alpha & -\alpha\sin 2\theta_{rm} + \beta\cos 2\theta_{rm} & \alpha\cos 2\theta_{rm} + \beta\sin 2\theta_{rm} \\ \hline -\alpha\cos\theta_{rm} + \beta\sin\theta_{rm} & \alpha\sin\theta_{rm} + \beta\cos\theta_{rm} & -\alpha\cos\theta_{rm} - \beta\sin\theta_{rm} & -\alpha\sin\theta_{rm} + \beta\cos\theta_{rm} \\ \alpha\sin\theta_{rm} + \beta\cos\theta_{rm} & \alpha\cos\theta_{rm} - \beta\sin\theta_{rm} & -\alpha\sin\theta_{rm} + \beta\cos\theta_{rm} & \alpha\cos\theta_{rm} + \beta\sin\theta_{rm} \end{array} \right] \tag{16c}
$$

$$\begin{bmatrix} F_\alpha \\ F_\beta \end{bmatrix} = M'_{24} \begin{bmatrix} \cos(\omega_2-\omega_4)t & \sin(\omega_2-\omega_4)t \\ -\sin(\omega_2-\omega_4)t & \cos(\omega_2-\omega_4)t \end{bmatrix} \begin{bmatrix} -(i_{ds}+i_{dr}) & i_{qs}+i_{qr} \\ i_{qs}+i_{qr} & i_{ds}+i_{dr} \end{bmatrix} \begin{bmatrix} i_{d's}+i_{d'r} \\ i_{q's}+i_{q'r} \end{bmatrix} \quad (27)$$

The first 2×2 matrix is a rotational coordinate matrix at angular frequency of $\omega_2-\omega_4$. As ω_2 and ω_4 are the angular frequency of 2-pole and 4-pole winding currents, respectively, it is shown that radial force is rotating at a speed of a difference of these angular frequencies. It simply means that radial force is modulated by revolving magnetic field.

For example, if 2-pole and 4-pole windings are motor and suspension windings, respectively, then, suspension currents are modulated by the motor currents in radial force generation. If the rotational direction of 4-pole magnetic field is opposite, then, radial force frequency is $\omega_2+\omega_4$. As the motor current frequency is adjusted to a shaft speed regulation, the rotational speed of radial force is regulated by ω_4. It is seen that radial force at any frequency and rotational directions can be generated by regulating an angular frequency of 4-pole winding currents.

Let us consider a case when ω_2 is equal to ω_4. Suspension winding current can be regulated to have exactly the same frequency of the motor winding currents. In this case, the first matrix is simplified, thus,

$$\begin{bmatrix} F_\alpha \\ F_\beta \end{bmatrix} = M'_{24} \begin{bmatrix} -(i_{ds}+i_{dr}) & i_{qs}+i_{qr} \\ i_{qs}+i_{qr} & i_{ds}+i_{dr} \end{bmatrix} \begin{bmatrix} i_{d's}+i_{d'r} \\ i_{q's}+i_{q'r} \end{bmatrix} \quad (27a)$$

In the equation (27a), $i_{ds}+i_{dr}$ and $i_{qs}+i_{qr}$ in the 2×2 matrix are the current components, those generate 2-pole airgap flux. Moreover, $i_{d's}+i_{d'r}$ and $i_{q's}+i_{q'r}$ are also the current components of 4-pole airgap flux. Therefore, it is found that the radial force is generated by an interaction of current components of 2-pole and 4-pole airgap fluxes.

In (27), the 2×2 matrix includes current components of the 2-pole airgap flux. The equation (27) is also written by a 2×2 matrix with 4-pole airgap flux current and 2×1 2-pole airgap flux current as,

$$\begin{bmatrix} F_\alpha \\ F_\beta \end{bmatrix} = M'_{24} \begin{bmatrix} -(i_{d's}+i_{d'r}) & i_{q's}+i_{q'r} \\ i_{q's}+i_{q'r} & i_{d's}+i_{d'r} \end{bmatrix} \begin{bmatrix} i_{ds}+i_{dr} \\ i_{qs}+i_{qr} \end{bmatrix} \quad (27b)$$

From radial force equations (27a)-(27b), the following considerations are given.

(a) Radial forces are generated by a product of airgap flux currents in 4-pole and 2-pole windings.

(b) The equation (27a) describes a relationship between radial forces and 4-pole winding currents in a case when the 4-pole winding is used as suspension winding. The inverse of the 2×2 matrix provides how suspension airgap flux currents should be regulated.

(c) The equation (27b) describes a radial force and current relationship when 2-pole winding is used as suspension winding. If pole-specific rotor short circuits are employed, then, 2-pole rotor current i_{dr} and i_{qr} are zero. Radial forces are proportional to the suspension stator winding currents.

If an airgap flux oriented vector controller is employed, then, $i_{q's}+i_{q'r}$ is zero. Then the 2×2 matrix is a diagonal matrix. Successful magnetic suspension is realized as reported in [6].

CONCLUSION

An analysis of radial forces considering rotor currents in the squirrel cage short circuit in an induction bearingless motor is clarified in the detail in this paper. It is shown that a product of airgap flux current components generates radial forces.

ACKNOWLEDGMENT

The authors would like to thank Mr. Yoshiteru Aikawa and Mr. Kenichi Kobayashi who were graduate students at the Tokyo University of Science.

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